City Size Distribution Dynamics in Transition Economies. A Cross-Country Investigation

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ABSTRACT. *The purpose of the present paper is to study the dynamics of the city size distribution in CEE and CIS transition economies, and identify the determinants of the variation of this distribution in time and across countries. We build a comprehensive unified database for CEE and CIS countries concerning city dynamics. We test the Gibrat`s law employing panel unit root tests that takes into account the presence of cross-sectional dependence and Nadaraya-Watson non-parametrical kernel regression. We construct a consensus estimate of the Pareto exponent of the city distribution using various econometric methods in order to investigate the fulfillment of Zipf`s law. We also test for non-Pareto behavior of the distribution when all the cities in a country are considered, using the Weber-Fechner law, the logarithmic hierarchy model, and the log-normal distribution. Not only we consider various distributions, but also study the "within distribution" dynamics by analyzing the individual cities relative positions and movement speeds in the overall distribution using a Markov chains methodology. In order to explain the differences in the city distributions and obtain valid statistical inference, we estimate, using cross-section dependence robust standard errors, a panel data fixed effects model to control for unobserved country specific determinants.*

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1 Introduction

The demise of the socialist economic system and its subsequent restructuring has led to profound changes in the spatial patterns of urban economies in cities of CEE and CIS. The most important and visible trend of urban development during the transition period has been the decentralization of economic activities, a process which has played a major part in the transformation of the post-socialist city. The privatization of assets and the introduction of land rent have been the two determinant factors governing the process

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of urban spatial readjustments within the reality of a new market-oriented social environment (Stanilov, 2007).

One of the most striking regularities in the location of economic activity is how much of it is concentrated in cities. Understanding urbanization and economic growth requires understanding the variety of factors that can affect the size of cities and their short-term dynamics. The existence of very large cities and the wide dispersion in city sizes are all particularly interesting qualitative features of urban structure worldwide. A surprising regularity, Zipf's law (Zipf, 1949) for cities, has itself attracted sustained interest by researchers over a long period of time. As early as Auerbach (1913), it was suggested that the city size distribution could be closely approximated by a *Pareto distribution* (power law distribution). City sizes are said to satisfy *Zipf's law* if, for large sizes S, we have $P(Size > S) = a / S^{\zeta}$, where a is a positive constant and $\zeta = 1$ (i.e. a power law distribution with unitary Pareto exponent). An approximate way of stating Zipf's law is the so-called rank size rule: the second largest city is half the size of the largest, the third largest city a third the size of the largest, etc. Zipf's Law can be related to another empirical regularity well known in urban economics. *Gibrat's Law* (Gibrat, 1931) states that the growth rate of an economic entity is independent of its initial size.

The purpose of this paper is to study the dynamics of the city size distribution in CEE and CIS transition economies, and identify the determinants of the variation of this distribution in time and across countries. More specifically we test empirically the validity of Gibrat's Law, compute a consensus estimate of the Pareto exponents of the city distribution for transition economies, test for non-Pareto behavior of the city size distribution, study the "within distribution" dynamics of individual cities in CEE and CIS economies using Markov chains, and identify, using cross-country data from CEE and CIS countries, the factors that drive the variation of the city distribution in these transition economies.

Taking into consideration the current state of knowledge, we extend the existing literature in several directions. First, we employ a battery of parametric and non-parametric tests for assessing the validity of Gibrat`s laws including panel unit root test robust to the presence of cross-sectional dependence. Second, we build a consensus estimate of the Pareto exponent of the city distribution in each country. Third, we will test for non-Pareto behavior using a wide range of alternative parametric distributions. Fourth, not only we will consider various distributions, but also study the "within distribution" dynamics by analyzing the individual cities relative positions and movement speeds in the overall distribution. Fifth, we employ a fixed effects model for assessing the determinants of city size distribution and ensure valid statistical inference using "robust" standard errors for cross-sectional dependence. Finally, we will build a new unified and comprehensive database for CEE and CIS countries consisting in city size data, as well as macroeconomic and socio-economic data that could explain the variation of the city size distribution.

The rest of the paper consists of five sections. In the first section we review the existing literature. In the following two sections we present the data employed in the study and we outline the methodology. In the forth section we discuss the results of our study and the final section concludes.

2 Literature Review

In the field of urban economics, Gibrat's Law and Zip`s Law has given rise to numerous empirical studies. In the 1990s numerous studies began to *test the validity of Gibrat's Law*, arriving at a consensus that it holds in the long term. Eaton and Eckstein (1997) concludes that considering only the 39 most populated French cities there is no correlation between city size and growth rate, accepting Gibrat's Law. This result goes against the one obtained by Guérin-Pace (1995) when considering a wide sample of cities with over 2,000 inhabitants. This is no surprising contradiction since Eeckhout (2004) demonstrates the importance of choosing sample size in the analysis of city size distribution: the arbitrary choice of a truncation point can lead to skewed results. However, Eaton and Eckstein (1997) and Davis and Weinstein (2002) accept the Gibrat's Law for Japanese cities, although they use different sample sections (40 and 303, respectively) and time horizons. Moreover, Davis and Weinstein (2002) argue that the effect of large temporary shocks (Allied bombing in the Second World War) on growth rates disappears completely in less than 20 years. Brakman et al. (2004), taking into consideration 103 German cities, concludes that bombing had a significant, but temporary impact on post-war city growth. Bosker et al. (2008) employs a sample of 62 cities in West Germany and finds evidence against Gibrat`s law for about 75% of the cites in the sample. Clark and Stabler (1991), using data panel methodology and unit root tests, accept the hypothesis of proportional urban growth for Canada. Resende (2004) accepts Gibrat`s law by applying the same methodology to 497 Brazilian cities. Ioannides and Overman (2003) accept the fulfillments of Gibrat's Law for the case of the US, taking into consideration a sample of 135 MSAs (Metropolitan Statistical Area). However, the hypothesis is rejected by Black and Henderson (2003) using a different set of MSAs.

These contradictory results may also be explained by the usage of different econometric methods. While Ioannides and Overman (2003) employs nonparametric techniques, Black and Henderson (2003) focuses mainly on panel data unit root tests. Eeckhout (2004) is the first study to use all the sample of cities in US, without size restrictions. Using both parametric and nonparametric methods, Eeckhout (2004) accepts Gibrat's Law for the US. For China, Anderson and Ge (2005) obtains a mixed result with a sample of 149 large cities. Petrakos et al. (2000) and Soo (2007) reject Gibrat's Law in Greece and Malaysia, respectively.

Recently, a reassessment of Gibrat's Law in the context of countries size and in the context of regions within a country has been carried out. González-Val and Sanso-Navarro (2010) finds evidence of Gibrat's Law if countries growth rates are considered. Giesen and Suedekum (2010) provides empiric evidence supporting the theory that Gibrat's law is satisfied not only at the aggregate national level, but also at the region level, showing that urban growth among large cities is scale independent basically "everywhere" in space in Western Germany.

A classical paper in the field of *testing the validity of Zip`s Law* is Rosen and Resnik (1980) who studied a cross section of 44 countries. They find that the Pareto coefficients differ across countries, ranging from 0.80 to 1.96 (e.g. Romania 1.085, Poland 1.127, Czechoslovakia 1.107, Hungary 1.092, USSR 1.278). Almost three-fourths of the countries have exponents significantly greater than unity. This indicates that populations in most countries are more evenly distributed than would be predicted by the rank-size rule. Soo (2005) updates Rosen and Resnik study using a cross-section of 73 countries and employs more robust econometric methods. The tests performed reject

Zipf's Law far more often than one would expect based on random chance. Also, the claim that Zipf's Law holds for urban agglomerations (Rosen and Resnick, 1980) is strongly rejected in favour of the alternative that agglomerations are more uneven in size than would be predicted by Zipf's Law. Roehner (1995) analyzes several countries, Eaton and Eckstein (1997) the cases of France and Japan, Brakman et al. (1999) the Netherlands, and Ioannides and Overman (2008) employs nonparametric procedures to study in detail the case of the United States.

These studies usually find the Pareto exponent for the US close to unity, but higher for most other countries. Several probabilistic and economic models have been proposed to account for this evidence. Among the most prominent probabilistic models are the ones by Gabaix (1999a, 1999b), and Cordoba (2008a, 2008b). Gabaix establishes that Gibrat's law can lead to Zipf's distributions if the number of cities is constant, but if new cities emerge only the upper tail is Zipf distributed. Cordoba (2008a) finds that a generalized Gibrat's law process, one that allows the variance, but not the mean of the city growth process to depend on city size, can account for Pareto exponents different from one even if the number of cities is constant. Cordoba (2008b) focuses on the more general case of an arbitrary exponent and derives conditions that standard urban models must satisfy in order to generate a balanced growth path and a Pareto distribution for the cities sizes.

There is an apparent contradiction in these studies, as they normally accept the fulfillment of Gibrat's Law but at the same time affirm that the distribution followed by city size is a Pareto distribution, very different to the lognormal (as implied by a process obeying Gibrat's Law). Eeckhout (2004) was able to reconcile both results, by demonstrating that imposing size restrictions on the cities (i.e. taking only the upper tail) skews the analysis. Thus, if all cities are taken, it can be found that the true distribution is lognormal, and that the growth of these cities is independent of size. Gonzalez-Val et al. (2008) confirm this result using the complete distribution of cities in US, Spain and Italy. In contrast to the success of the probabilistic approach, most of the economic models have failed to match the evidence. Krugman (1996) points out that none of the existing economic models can explain the data. Recently, Rossi-Hansberg and Wright (2007) construct a stochastic urban model along the lines of the deterministic model of Black and Henderson (1999). Like Black and Henderson, they are able to produce proportional growth, and Zipf distributions only under particular restricting conditions. Numerical simulations confirm that large cities in their model are too small compared with the predictions of a Zipf distribution, suggesting a Pareto exponent different from unity, or the possibility that the distribution is non-Pareto as suggested by Parr and Suzuki (1973) and Eeckhout (2004).

While obtaining the value for the Pareto exponent for different countries is interesting in itself, there is also of great importance to *investigate the factors that may influence the value of the exponent*, for such a relationship may point to interesting economic and policy-related issues. The Pareto exponent can be viewed as a measure of inequality: the larger the value of the Pareto exponent, the more even is the populations of cities in the urban system. There are many potential explanations for this variation. One of them relies on economic geography models (i.e. Krugman, 1991), models that can be interpreted as models of unevenness in the distribution of economic activity. The key parameters of these models are the degree of increasing returns to scale, transport costs, size of industrial sectors, and size of external trade. There will be a more uneven distribution of city sizes (smaller Pareto exponent), the greater are *scale economies*, the lower the *transport costs*, the smaller the *share of manufacturing* in the economy, and the lower the *share of international trade* in the economy. Rosen and Resnick (1980), find that the Pareto exponent is positively related to *per capita GNP*, *total population* and *railroad density*, but negatively related to *land area*. Mills and Becker (1986), in their study of the urban system in India, find that the Pareto exponent is positively related to total population and the *percentage of workers in manufacturing*. Alperovich (1993) cross-country study finds that it is positively related to per capita GNP, population density, and land area, and negatively related to the *government share of GDP*, and the *share of manufacturing value added in GDP*. This study also finds that Pareto exponent first decreases and then increases with per capita GNP when the country goes through different phases of development. There may also be *political factors* that could influence the size distribution of cities. Ades and Glaeser (1995) argue that *political stability* and the *extent of dictatorship* are key factors that influence the concentration of population in the capital city. They conclude that political instability or a dictatorship should imply a more uneven distribution of city sizes. Soo (2005) finds that political variables have more explanatory power of the variation than economic variables. All the four variables in Rosen and Resnick (1980) plus the size of non-agricultural sectors, the size of international trade, and the degree of scale economy either are insignificant or enter with opposite sign to what theoretical models would predict. The investigation also finds that the size of government expenditure is positively related to Pareto exponent, which contradicts Alperovich (1993). Jiang et al. (2008) empirically explores the relationship between city size distribution and economic growth, based on a panel data analysis using China provincial data from 1984 to 2005 capturing the idea that *government intervention on labor migration* distorts city size distribution. Also, improvements in *information and communication technologies (ICT)* may lead to changes in urban structure, for example, because they reduce the costs of communicating ideas from a distance. In a recent paper, Ioannides et al. (2008) examines the effects of ICT on urban structure and find robust evidence that increases in the *number of telephone lines per capita* and the *number of internet users* encourage the spatial dispersion of population in that they lead to a more concentrated distribution of city sizes. They develop a model predicting that *macroeconomic volatility* influences the city distribution, but they find no empirical support.

3 Data

The analysis in this paper in based on a new, unified and comprehensive database for CEE and CIS countries consisting in city size data, as well as macroeconomic and socio-economic data that could explain the variation of the city size distribution. In this section we describe the data collected so far that is in different stages of processing.

It is obvious that studying the dynamics of the city distribution gives more precise results if one employs a larger sample of cities, towns and villages. However, there is a trade-off between the size of the sample and the frequency of the data in that sample. Therefore, we have built two data sets. The first one consists on data, with annual frequency, on cities over 100,000 inhabitants. The second one is focused on detailed city size data, but with the time spans and the frequencies different for each of the country.

Regarding the *cities over 100,000 inhabitants* for the time span 1970 - 2007, the main source of the data is the annual United Nations Demographic Yearbooks (UNDY). The main difficulty consisted in reconstructing the data backwards, before 1989, on cities in the Former USSR countries since they are reported under USSR. The situation is similar for some of the CEE countries, such as the countries in the Former Yugoslavia, or the Czech Republic and Slovakia. To ensure that the database has a reduced number of missing observations we have collected data no mater the methodology employed in UNDY in different years (i.e. CDJC - census de jure, complete tabulation; ESDF - estimates, de facto; ESDJ - estimates, de jure). The number of cities over 100,000 in the CEE-CIS region is reported in Table 3.1.

Table 3.1. Number of cities over100,000 inhabitants in CEE-CIS countries for 1970 - 2007

			average	min	max
1	Albania	CEE	1.00	1	1
2	Armenia	CIS	2.92	2	3
3	Azerbaijan	CIS	3	3	3
4	Belarus	CIS	11.71	9	15
5	Bosnia and Herzegovina	CEE	3.95	1	7
6	Bulgaria	CEE	7.95	4	10
7	Croatia	CEE	3.71	3	4
8	Czech Republic	CEE	6.03	4	8
9	Estonia	CEE	1.79	1	2
10	Georgia	CIS	4.55	4	5
11	Hungary	CEE	8.24	6	9
12	Kazakhstan	CIS	18.14	15	20
13	Kyrgyz Republic	CIS	2	2	2
14	Latvia	CEE	2.16	2	3
15	Lithuania	CEE	4.39	3	5
16	Macedonia, FYR	CEE	1.03	1	2
17	Moldova	CIS	3.13	2	4
18	Poland	CEE	37.08	23	43
19	Romania	CEE	21.03	13	26
20	Russian Federation	CIS	149.68	124	179
21	Serbia	CEE	8.03	2	21
22	Slovak Republic	CEE	2.00	2	2
23	Slovenia	CEE	1.63	1	2
24	Tajikistan	CIS	1.89	1	2
25	Turkmenistan	CIS	2.14	1	3
26	Ukraine	CIS	44.50	39	51
27	Uzbekistan	CIS	13.4	8	17

The data on cities over 100,000 inhabitants is employed for analyzing the validity of Gibrat Law and for estimating the Pareto coefficient of the city size distribution.

Regarding the *detailed city data*, the main source are the national official statistical information services of CEE and CIS countries. Table 3.2 presents the detailed data we have acquired so far and that is in various stages of processing.

Table 3.2. Detailed of city data for CEE-CIS countries

		Period	Level of detail
1	Armenia	1989, 2002, 2008	all cities
2	Azerbaijan	1979, 1989, 2002, 2010	all cities
3	Belarus	1989-2009	all cities
4	Georgia	1989, 2002, 2009	all cities
5	Hungary	1970, 1980, 1990, 2001	all cities and villages
		1870, 1880, 1890, 1900, 1910, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990, 2000	all cities
6	Kyrgyz Republic	1989, 1999	all cities
7	Latvia	1990 - 2009	all cities
8	Poland	2004 - 2009	all cites
9	Romania	1991, 2002	all cities and villages
10	Russian Federation	1996 - 2004	all cities
11	Serbia	1991, 2002	all cities
12	Slovenia	1981, 1991, 2002	all cities
13	Tajikistan	1989, 1999, 2006	all cities
14	Turkmenistan	1989, 1995, 2006	all cities
15	Ukraine	1989, 2001 2008	all major cities
16	Uzbekistan	1991, 2002, 2006	all cities

The detailed city data is employed for analyzing the validity of Gibrat Law, for estimating different parametric repartition functions for the city size distribution, and for analyzing the "within distribution" city dynamics using Markov chains.

Macroeconomic and socio-economic cross-country data is employed in order to determine the factors that influences of city size distribution. The main sources of data for this database are World Bank World Development Indicators, Penn World Table, IMF International Financial Statistics, International Road Federation World Road Statistics, OECD Telecommunications and Internet Statistics, OECD International Regulation Database, and national official statistical information services of CEE and CIS countries.

4 Methodology

4.1 Testing the validity of Gibrat's Law

The Gibrat`s law hypothesis is tested by employing both parametric and nonparametric methods. The simplest parametric test consists in estimating the following growth equation:

$$\ln S_{it} - \ln S_{it-1} = \alpha + \beta \ln S_{it-1} + \varepsilon_{it} \tag{1}$$

where S_{it} denotes the size of city i at the time t. Gibrat`s law holds if $\beta = 0$ (i.e. growth is independent of the initial size). To ensure validity of the statistical results one must adjust the standard errors of the coefficient estimates for possible dependence in the residuals. The results of these regressions are usually heteroskedastic (Gonzalez-Val et al., 2008), so it is suggested in the literature to compute the standard errors using White Heteroskedasticity-Consistent Covariance Matrix Estimator (White, 1980). However, another question to be tackled is the presence of cross-sectional dependence in panel data on city sizes. The cross-sectional dependence is tested using the Pesaran (2004) test, which does not depend on any particular spatial weight matrix when the cross-sectional dimension is large. In this paper, to account for the effect of potential cross-correlated residuals, Driscoll and Kraay (1998) standard errors are employed, Driscoll and Kraay (1998) modifies the standard Newey and West (1987) covariance matrix estimator such that it is robust to very general forms of cross-sectional as well as temporal dependences. Moreover, it is suitable for use with both, balanced and unbalanced panels (Hoechle, 2007).

Clark and Stabler (1991) pointed out that testing for Gibrat's Law is equivalent to testing for the presence of a unit root. This idea has also been emphasized by Gabaix and Ioannides (2004). If the null hypothesis that the city population time series has a unit root is rejected, the null hypothesis that its size evolves according to Gibrat's Law is also rejected. Panel data unit root tests have been proposed as alternative, more powerful tests than those based on individual time series unit roots tests. The panel unit root approach to investigate the validity of Gibrat`s Law has been pioneered by Clark and Stabler (1991) and has already been applied by Davis and Weinstein (2002), Resende (2004), Henderson and Wang (2007), Soo (2007) and Bosker et al. (2008).

Also, when exploring the existence of unit roots in panel data, it is important to take into account the presence of cross-sectional dependence. Most of these studies employed conventional (*i.e.* first generation) unit root tests that assume cross-sectional independence. The first generation test proposed by Levin, Lin and Chu (2002) is applicable for homogeneous panels where the coefficients for unit roots are assumed to be the same across cross-sections. Im, Pesaran and Shin (2003) allows for heterogeneous panels and proposes panel unit root tests which are based on the average of the individual ADF unit root tests computed from each time series. The null hypothesis is that each individual time series contains a unit root, while the alternative allows for some but not all of the individual series to have unit roots. However, the correct application of these techniques depends crucially on the assumption that individual time series are cross-sectional independent. This might be a restrictive assumption when using city size panel data. Conventional panel unit root tests, such as Levin, Lin and Chu (2002) and Im, Pesaran and Shin (2003), could lead to significant size distortions in the presence of neglected cross-section dependence and, generally, to over-rejection of the null hypothesis.

Much of the recent research on non-stationary panel data has focused on the problem of cross-sectional dependence. Second generation panel unit root tests that take into account the potential cross-section dependence in the data have been developed; see the recent survey by Breitung and Pesaran (2008). A number of panel unit root tests that allow for cross section dependence have been proposed in the literature that use orthogonalization type procedures to asymptotically eliminate the cross dependence of the series before standard panel unit root tests are applied to the transformed series (Bai and Ng, 2004; Moon and Perron, 2004). On the other hand, Pesaran (2007) suggests a simple way of accounting for cross-sectional dependence. This method is based on augmenting the usual ADF regression with the lagged cross-sectional mean and its first difference to capture the cross-sectional dependence that arises through a single-factor model. The proposed test has the advantage of being simple and intuitive. It is also valid for panels where the cross-sample dimension (N) and the time dimension (T) are of the same orders of magnitudes. The Monte Carlo simulations employed by Pesaran (2007) suggests that the panel unit root tests have satisfactory size and power even for relatively small values of *N* and *T* (i.e. 10<N<200 and 10<T<200).

The present study makes use of a battery of first and second generation panel unit root tests. More specifically we employ the first generation Levin, Lin and Chu (2002) and Im, Pesaran and Shin (2003) tests, and the second generation Pesaran (2007) test.

In order to increase the robustness of the results, nonparametric tests are also implemented. As suggested by Ioannides and Overman (2003) and Eeckhout (2004) for the non-parametrical analysis of Gibrat's law it is better to use normalized city growth rates (*i.e.* from growth rate of city *i* in year *t* the mean is subtracted and the result divided by the standard deviation of the growth rates). The widely employed Nadaraya-Watson kernel regression technique (Nadaraya, 1964, 1965; Watson 1964; Hardle, 1992) establishes a functional form-free relationship between population growth and country size for the entire distribution. It consists of taking the following specification:

$$g_i = m(s_i) + \varepsilon_i \tag{2}$$

where g_i stands for the normalized growth of city i, and s_i is the logarithm of its size. Therefore, instead of assuming a linear relationship between these two variables, as in equation (1), $m(\cdot)$ is estimated as a local average, using a kernel function $K(\cdot)$:

$$m_{NW}(s) = \frac{\frac{1}{n}\sum_{i=1}^{n} K\left(\frac{s-s_i}{h}\right) g_i}{\frac{1}{n}\sum_{i=1}^{n} K\left(\frac{s-s_i}{h}\right)} \tag{3}$$

where n is the sample size, and h the kernel bandwidth.

Starting from the estimated mean, $m_{NW}(\cdot)$, the variance of the growth rate can also be computed using the corresponding Nadaraya-Watson estimator:

$$\sigma^2_{NW}(s) = \frac{\frac{1}{n}\sum_{i=1}^{n} K\left(\frac{s-s_i}{h}\right) (g_i - m_{NW}(s))^2}{\frac{1}{n}\sum_{i=1}^{n} K\left(\frac{s-s_i}{h}\right)} \tag{4}$$

Under the null of urban growth independent of initial size one would expect that all cities, regardless of their size, have mean normalized growth rate equal to zero and variance equal to one. These hypotheses are tested by constructing bootstrapped 95-

percent confidence bands, calculated from 500 random samples with replacement, as suggested by González-Val and Sanso-Navarro (2010).

The nonparametric techniques employed in this paper allows computing a variety of nonparametric and semi-parametric kernel-based estimators appropriate for a mix of continuous, discrete, and categorical data (Hayfield and Racine, 2008). This kind of non-parametric technique is convenient because it allows identifying the influence of discrete variables accounting for possible structural breaks. The basic idea underlying the treatment of kernel methods in the presence of a mix of categorical and continuous data lies in the use of generalized product kernels. Li and Racine (2003) proposed the use of these generalized product kernels for unconditional density estimation and developed the underlying theory for a data-driven method of bandwidth selection for this class of estimators. The use of such kernels offers a seamless framework for kernel methods with mixed data. Further details on a range of kernel methods that employ this approach can be found in Li and Racine (2007). When all the variables are continuous, these methods collapse to the familiar Nadaraya-Watson nonparametric regression estimators.

The default Gaussian kernel is employed since the specific form of the local averaging function does not have a major impact on the results. On the other hand, bandwidth selection is a key aspect of sound nonparametric kernel regression estimators. The basic approach in the related urban literature (Eckhout, 2004) is to compute the bandwidth according to the "rule of thumb" proposed by Silverman (1986) based on inter-quartile range. In the present study, the bandwidth is selected using a data-driven method, more specifically, the Kullback - Leibler cross-validated bandwidth selection, using the method of Hurvich et al. (1998).

4.2 Estimating the Pareto exponent of the city size distribution

The most communally used parametric estimation procedure of the Pareto exponent is the so called Zipf regression, i.e. regressing the logarithm of the rank of a city on the logarithm of its size. One potentially serious problem with the Zipf regression is that it is biased in small samples. Gabaix and Ioannides (2004) show, using Monte Carlo simulations, that the coefficient of the Zipf regression is biased downward for sample sizes in the range that is usually considered for city size distributions and that OLS standard errors are grossly underestimated. Therefore, in this paper we employ a consensus estimate (Graybill and Deal, 1959) of the Pareto exponent using two alternative econometric methods. The consensus estimate will be weighted with the inverse of the standard errors of the estimates from the two methods. The first method (Gabaix and Ibragimov, 2009) consists in a modified Zipf regression:

$$\ln\left(R_{it} - \frac{1}{2}\right) = a - \zeta \ln(S_{it}) + \varepsilon_t \tag{5}$$

where R_i is the rank of city i in year t. Gabaix and Ibragimov (2009) argue that the shift of 0.5 is optimal, and reduces the bias to a leading order. They show that the standard error on the Pareto exponent ζ is not the OLS standard error, but is asymptotically $\left(2/n\right)^{1/2}\zeta$.

The second method, developed in Gabaix and Ioannides (2004) and also employed by Soo (2005) consists in calculating the value of the Pareto exponent using the Hill estimator:

$$\zeta_H = \frac{n-1}{\sum_{i=1}^{n-1}\left(\ln S_{(i)} - \ln S_{(n)}\right)} \tag{6}$$

Under the null hypothesis of the power law, the Hill estimator is the maximum likelihood estimator, and it is therefore asymptotically efficient.

4.3 Testing for non-Pareto behavior of the city size distribution

First, as suggested by Rosen and Resnick (1980) we will *test for non-Pareto behavior* by include higher order terms of the logarithm of city size in the Zipf regression:

$$\ln(R_{it}) = a - \zeta \ln(S_{it}) + b\ln^2(S_{it}) + c\ln^3(S_{it}) + \varepsilon_t \tag{7}$$

and test the statistical significance of their coefficients. However, we must be cautious of the results, since Gabaix and Ioannides (2004) show that, even if the actual data exhibit no nonlinear behavior, OLS regression of (7) will yield a statistically significant coefficient for the quadratic term 78% of the time in a sample of 50 observations.

Estimation of parameters in the OLS regression (5) of the logarithm of shifted rank of the cities on the logarithm of their size will be conducted for 20, 10 and 5 percentage tails of the sample of all cities of the country. This will further allow us to determine the smallest critical quantity of population for cities in different countries considered, where Zipf's Law begins to hold.

We will also consider the non-Pareto behavior of the city size distribution using alternative parametric models such as the *Weber-Fechner Law*, whose parameters can be estimated by using the regression:

$$\ln(S_{it}) = \beta - \gamma R_{it} + \varepsilon_t \qquad (8)$$

where the coefficient γ is the so called Weber's constant, which shows how the size changes with the change in the rank. In case of the Weber-Fechner law, the rank of the city changes in arithmetic progression with the change of the size of the city in geometric progression, while in case of the Zipf's law both rank and the size of the city change in arithmetic progression.

In general, Zipf's law does not hold for small cities (with the size below a cut-off). Therefore, we expect that the Weber-Fechner law would better describe the *whole sample of all cities* and other populated areas in a certain country. It should also be noted that in terms of statistical characteristics one natural extension of the Weber-Fechner law is a *logarithmic hierarchy model*:

$$\ln i = c + \alpha_1 \ln N_i + \alpha_2 \ln_2 N_i + \alpha_3 \ln_3 N_i + \alpha_4 \ln_4 N_i \qquad (9)$$

where $\ln_k y$ denotes the *k*th iteration of logarithm (i.e. $\ln_k y = \underbrace{\ln \ln \ldots \ln}_{k} y$, k≥1).

The authors will further focus on other distributional alternatives, including *the log-normal distribution* that was used in several studies to describe the distribution of all cities in a country (Eeckhout, 2004; Gonzalez-Val et al., 2008). Using several distribution goodness-of-fit tests (e.g. Kolmogorov-Smirnov, Anderson-Darling) we will determine the optimal distributional models for the analyzed city size data.

4.4 Studying the "within distribution" city dynamics

Zipf's and other distribution laws allow the characterization of the evolution of the global distribution, but they do not provide any information about the movements of the towns within this distribution. A possible way to answer these questions is to track the evolution of each city's relative size over time by estimating transition probability matrices associated with *discrete Markov chains*. This line of analysis has first been pursued by Eaton and Eckstein (1997) and then by Black and Henderson (2003).

We assume that the frequency of the distribution follows a first-order stationary Markov process. In this case, the evolution of the city size distribution is represented by a transition probability matrix, M, in which each element (i, j) indicates the probability that a city that was in class i at time t ends up in class j in the following period. The way of cities' division on classes will be chosen by considering the performance of the test for Markovity of order one. Then each element p_{ij} of the transition matrix is estimated as a conditional probability $p_{ij} = P(A_j(t+1) \mid A_i(t))$, where $A_i(t)$ is the event that "city is in a state i at time t". In other words we find shares of cities remained in each size class at the end of the period and moved up or down by the end of the period. Denoting by $F_t = \begin{pmatrix} p_1(t) & p_2(t) & \ldots & p_k(t) \end{pmatrix}$ the vector of probabilities that a city is in class i at time t, the dynamics of this vector is given by:

$$F_{n+1} = F_n M = F_0 M^{n+1} \tag{10}$$

Next, we determine the ergodic distribution that can be interpreted as *the long-run equilibrium* city-size distribution. Explicitly, given that the transition matrix M is regular, then M^n tends to a limiting matrix M^* when n tends to infinity (Kemeny and Snell, 1960). Therefore, with the passage of time, the distribution of cities will not change any more and will converge to the ergodic or limit distribution. Concentration of the frequencies in a certain class would imply convergence (if it is the middle class, it would be convergence to the mean), while concentration of the frequencies in some of the classes, that is, a multimodal limit distribution, may be interpreted as a tendency towards stratification into different convergence clubs. Finally, a dispersion of this distribution amongst all classes is interpreted as divergence.

We also determine the *speed of the movement of a city within the distribution*, using the mean first passage time matrix M_P, that can be easily constructed for the transition matrix M (Kemeny and Snell, 1976). The (i,j) element of the matrix M_P indicates the expected time for a city to move from class i to class j for the first time.

Thus, using Markov chains we can perform a more complete analysis of movement speed and form of convergence within the city size distribution.

4.5 Identifying the factors that drive the variation of the city distribution

We follow Rosen and Resnick (1980) and Soo (2005), but we also exploit the panel structure of the data to control for unobserved country specific determinants of differences in the city size distribution. Thus, we estimate a *fixed effects model* (Baltagi, 2005; Hsiao, 2003):

$$\zeta_{it} = \mu_i + \alpha_t + \beta X_{it} + \varepsilon_{it} \tag{11}$$

where ζ_{it} is the consensus estimate of the Pareto exponent for the country i at time t, μ_i is a country specific constant, α_t is a time specific constant, and X_{it} a collection of explanatory variables that are supposed to determine the city size distribution: economic geography variables, political variables, ICT variables, socio-economic variables.

As described in the literature review section, the results concerning the direction and the amplitude of the factors that influences the distribution are quite contradictory. These mixed results may be due to inappropriate estimation methods. Soo (2005) suggests that using an estimated coefficient as a dependent variable in a regression, might lead to inefficient estimates of the regression coefficients due to induced heteroskedasticity. As it is well known (e.g. Wooldridge, 2001), if the residuals are not spherical the significance tests computed using OLS standard errors are not valid and, therefore, the inference based on this tests can be misleading. To ensure validity of the statistical results one must adjust the standard errors of the coefficient estimates for possible dependence in the residuals. However, according to Petersen (2007) a substantial fraction of published articles in leading journals fail to adjust the standard errors when using panel data models. Although most studies provide standard error estimates that are consistent when heteroscedasticity and autocorrelation is present, cross-sectional dependence is still largely ignored. Parks (1967) and Kmenta (1986) proposed a feasible generalized least squares (FGLS) based algorithm to account both for heteroscedasticity as well as for temporal and spatial dependence in the residuals of panel data models, However, Beck and Katz (1995) pointed out that the Parks-Kmenta method tends to produce unacceptably small standard error estimates, and they introduced the method of panel corrected standard errors (PCSE). Soo (2005) in his cross-country study on city size distributions advocates the use OLS coefficient estimates with panel corrected standard errors. Nevertheless, Driscoll and Kraay (1998) and Hoechle (2007) points out that the finite sample properties of the PCSE estimator are rather poor when the panel's cross-sectional dimension *N* is large compared to the time dimension *T*. Driscoll and Kraay (1998) demonstrate that this problem can be solved by modifying the standard Newey and West (1987) covariance matrix estimator such that it is robust to very general forms of cross-sectional as well as temporal dependences. Moreover, it is suitable for use with both, balanced and unbalanced panels. In this paper we employ Driscoll-Kraay standard errors in order to ensure valid statistical inference

Following Ioannides et al. (2008), in order to ensure the robustness of the results, we intent to employ other measures of urban concentration as dependent variable in equation (11): the coefficient of variation, the Gini index, and the normalized Herfindahl concentration index. These measures, that are computed using the consensus estimate of the Pareto exponent, reflect different aspects of dispersion.

5 Results

5.1 Results concerning Gibrat Law

In this section Gibrat`s law is investigated using two datasets of cites from transition economies. The first dataset consists in detailed city size data from Poland, Belarus and Latvia for the period 2000-2009. More specifically, in the case of Poland the largest 200 cities are considered, in Belarus the largest 50 cities, and in Latvia the largest 30 cities. The main source of the detailed data is the national official statistical information services of the respective countries. The second dataset is focused on data for the period 1970 – 2007 on cities over 100,000 inhabitants from twelve transition economies, namely Russia, Ukraine, Poland, Romania, Belarus, Bulgaria, Hungary, Czech Republic, Slovak Republic, Estonia, Latvia and Lithuania.

Five of the countries are pooled into two groups, since there is a relatively low cross-section dimension when analyzed separately. The first group consists of the Baltic States (Estonia, Latvia, Lithuania), the second one of the countries from the Former Czechoslovakia (Czech Republic, Slovak Republic). The average number of cities over 100,000 inhabitants for the remaining units is as follows: Russian Federation 152, Ukraine 45, Poland 37, Romania 21, Belarus 12, Bulgaria 8, Hungary 8, Former Czechoslovakia 8, and Baltic States 8.

Table A.5.1.1 in the Appendix describes the dataset, presenting the number of observations, the time and cross-section dimensions of the panel, the average, standard deviation, minimum and maximum city size.

5.1.1 Gibrat`s law for detailed city data

In this subsection the analysis is conducted on the dataset containing detailed city size data in Poland, Belarus and Latvia for the period 2000 – 2009. Pooling observations and using panel data methods is a necessary strategy to increase the reliability of the estimates when the observed period is relatively short (Banerjee, 1999). First, the growth equation (1) was estimated using both pooled data and a fixed effects panel model. The results of these estimations are presented in the first two lines of Table 5.1.1. In the urban literature, to test the significance of the parameters, White (1982) standard errors are generally employed since they are robust to heteroskedastic innovations. However, in this case, the estimated regression residuals of the fixed effects model are cross-sectionally dependent, as is clearly noticeable in the third line from Table 5.1.1. The pair-wise cross-section correlations coefficients of the residuals are not zero, since the average absolute correlation between the residuals of two cities is 0.318 in Poland, 0.39 in Belarus, and 0.341 in Latvia. Also, Pesaran (2004) cross-sectional dependence test rejects the null hypothesis of spatial independence on any standard level of significance. Therefore, this finding indicates that it is advisable to test for significance using Driscoll and Kraay (1998) standard errors, since they are robust to very general forms of cross sectional and temporal dependence.

Table 5.1.1. *Results for detailed city data in Poland, Belarus and Latvia*

	Poland	Belarus	Latvia
ln(Size)	-0.0011	0.0029	0.0006
pooled	[0.0001]	[0.0004]	[0.0003]
	(0.0000)	(0.0000)	(0.0550)
ln(Size)	-0.0063	-0.0827	-0.1423
fixed effects	[0.0076]	[0.0475]	[0.0770]
	(0.4030)	(0.0880)	(0.0750)
ACSC	0.3180	0.3900	0.3410
PCS	34.6650	24.2510	7.6140
	(0.0000)	(0.0000)	(0.0000)
HWH	25.0400	27.7400	9.1400
	(0.0000)	(0.0000)	(0.0053)
URLLC	-0.0026	-0.6400	-3.2343
	(0.4989)	(0.2610)	(0.0006)
URIPS	10.8370	4.5420	1.4160
	(1.0000)	(1.0000)	(0.9220)
URPCS	-0.0060	-0.6400	-0.3220
	(0.4980)	(0.2610)	(0.3740)

Driscoll - Kraay robust standard errors are reported in squared parentheses; p-values are reported in round parentheses; ACSC is the average absolute value of the off-diagonal elements of the correlation matrix of the regression residuals; PCS is the Pesaran (2004) cross-section independence test; HWH is the modified Hausman (1978) test; URLLC, URIPS, URPCS are Levin et al (2002), Im et al (2003) and Pesaran (2007) panel unit root tests; the transformed t statistics are reported for the unit root tests

The estimates of the pooled model provide strong evidence for the rejection of Gibrat`s law in Poland and Belarus. The evidence in the case of Latvia is less clear since the null hypothesis that the parameter connecting the growth rate and the size of a city is zero can be rejected at a level of significance of 5%, but not at a level of significance of 1%. These findings are consistent with the results of the non-parametric estimations, presented in Figure A.5.1.1 in the Appendix. This is no coincidence, since the non-parametric technique is an alternative estimation method of the pooled model.

However, one has to be careful when pooling the data since this can invalidate the analysis. For example, if the true model is fixed effects, the pooled OLS yields biased and inconsistent estimates of the regression parameters (Baltagi, 2005). In order to test for the presence of cross-section specific fixed effects, it is common to perform a Hausman (1978) test. In this paper, the null hypothesis of no fixed effects is tested using a version of the Hausman (1978) test proposed by Wooldridge (2001) and Hoechle (2007). Since this version of the test is robust to very general forms of spatial and temporal dependence it should be suitable for the case of city size panel data. The results of the tests are presented in the fourth line of Table 5.1.1. They provide strong evidence in the favor of the fixed effects model because the null of no fixed effects is rejected at any usual level of significance. The estimates from the fixed effects model provide contrary evidence to that indicated by the pooled data model. As it turns out, when accounting for city specific effects, the null hypothesis of cities growing independent of their size can not be rejected at the level of 5% for any of the three countries.

Next, the panel structure of the city population data is further exploited in order to test for a unit root. Although only 10 observations over time are available, the use of a panel unit root test with a relatively large cross-section dimension is likely to alleviate the small-sample bias of a usual ADF unit root test. Black and Henderson (2003) also employs 10 time observation (decade by decade) in their study on urban evolution in the USA. Following Clark and Stabler (1991) only a constant has been included as the deterministic term. The results for the first generation Levin, Lin and Chu (2002) and Im, Pesaran and Shin (2003) tests, and the second generation Pesaran (2007) test are reported in the last three lines of Table 5.1.1.

Although, the first generation tests are used for completeness, more weight is given to the test of Pesaran (2007) since it allows investigating the presence of a unit root taking into account cross-sectional dependence, which is the case of the analyzed sample. Moreover, the test is robust to size distortions caused by the potential presence of serially correlated errors. As one can easily notice, the test can not reject the null of a unit root at any usual level of significance, therefore, providing support for the acceptance of Gibrat`s law in all the three countries.

However, it has to be stressed that, since specific city effects are taken into account, the deterministic component (the expected growth rate) is different across cities. Therefore, although the coefficient that quantifies the influence of the size on growth is zero, a consistent difference in the expected growth rate between "small" cities and "large" cities might indicate that Gibrat`s law does not hold. This could be the case of Belarus, because the non-parametric analysis indicates that there are differences between the behavior of small cities, medium cities and large cities.

To investigate further, the cities in Belarus are grouped in three categories, respectively the "large" cities group consisting of the largest 8 cities, the "medium" group comprising the next largest 27 cities, and the "small" group with the last 15 cities. The grouping was done such that the modified Hausman (1978) test indicates that for each of the group a pooled model is adequate. There is a significant difference between the average growth rates of the cities in these groups, with an average annual growth of 0.49% for the first group, -0.15% for the second group, and -0.46% for the small cities group. Therefore, a growth regression was estimated for each of the group, and another one for the entire sample but controlling for group specific characteristics. The results are reported in Table A.5.1.2 in the Appendix. It seems that for the large cities group there is a significant dependence of growth on size. Moreover, after the dummy variables controlling for different groups are accounted for, the coefficient quantifying the dependence of the size of the city on its growth rate is statistically significant at 5%. This finding proves the validity of intuitive doubts as to proportionality of growth in Belarus where the intentionally designed redistribution measures are evident.

Overall, in the period 2000-2009 there is very strong evidence that Gibrat`s law holds for Latvia and strong evidence that in is valid in Poland. However, it seems that, at least in the short run, there is a divergence pattern in the case of Belarus. A longer time span is necessity for a deeper investigation of the long run dynamics of city growth.

5.1.2 Gibrat`s law for cities over 100,000 inhabitants

In this subsection the analysis turns to cities over 100,000 inhabitants in the period 1970 – 2007. There are twelve countries in the sample, but, after pooling some of them as described above, nine units remain, respectively Russia, Ukraine, Poland, Romania, Belarus, Bulgaria, Hungary, Former Czechoslovakia, and Baltic States.

A major problem with this dataset is the existence of missing observations. Although, data were collected irrespective of the methodology employed in the UNDY in different years, Hungary is the only country in the sample that has all the 38 observations over time. In the Baltic States there are 32 time observations, in Bulgaria 28, in Belarus and Poland 27, in Romania 26, in Former Czechoslovakia 25, in Russia 24, and in Ukraine only 17. Moreover, since growth rates are needed in our analysis, the problem of missing data is further amplified since the growth rate can not be computed if consecutive year data is not available. When estimating the growth regression using pooled data or the fixed effects model, an assumption had to be made in order to alleviate this problem of missing growth rates. More specifically, if city sizes data is missing in year *t*, but not in year *t-1*, the growth rate of a city for the period *t/t-1* is, however, computed by assuming to be equal to the annual average growth rate between year *t* and the year with the next available city sizes data. This is a reasonable assumption since it does not introduce new city data by interpolation. It uses only the original city size data, but it computes the growth rates with different formulas depending on the situation.

First, the growth equation (1) was estimated using both pooled data and a fixed effects panel model. To capture the influence of the breakdown of the communist regime the sample is also divided in two subsamples, respectively 1970-1989 and 1990-2007. The results are reported in Table 5.1.2. The null of no fixed effects can not be rejected at the level of significance of 1% for any of the countries. Although, the results of the fixed effects model are reported for completeness, more weight should be, therefore, given to the pooled model in this case. To ensure that the panels are balanced some of the cities with sparse observations were drooped. Therefore, the number of analyzed cities is 108 for Russia, 31 for Ukraine, 23 for Poland, 13 for Romania, 9 for Belarus, and 6 for Bulgaria, Hungary, Former Czechoslovakia and the Baltic States. The average absolute value of the off-diagonal elements of the correlation matrix of the regression residuals varies from 31.7% for Poland to 72.6% for Romania. Also, the null hypothesis of cross-sectional independence is rejected for all the countries, implying the necessity of using Driscoll and Kraay (1998) standard errors to correct for cross sectional dependence.

The results of the pooled regression indicates that, in the post-communist period, Gibrat`s law is valid in all of the countries, with some doubts in the case of Hungary. When all the sample is considered the evidence for accepting Gibrat`s law is less clear in Russia, Ukraine, Poland, and Romania. These findings are largely confirmed by the results of the non-parametrical regressions that are provided in Table A.5.1.2 in the Appendix. However, these results indicate that there is strong support for the law of proportional effect in the case of Russia and Ukraine, when the entire sample is considered.

Table 5.1.2.. *Growth regressions results for cities over 100,000 inhabitants for the period 1970-2007*

		Pooled regression			HWH	Fixed effects regression			ACSC	PCS	
		estim.	std. err.	p-value		estim.	std. err.	p-value		statistic	p-value
Russia	all sample	-0.0060	0.0027	0.0265	5.7100	-0.2052	0.0655	0.0022	0.3800	100.58	0.0000
	before 1989	-0.0036	0.0010	0.0003	(0.0186)	-0.1499	0.0633	0.0196	0.6350	135.03	0.0000
	after 1989	-0.0065	0.0044	0.1418		-0.4061	0.0591	0.0000	0.4910	38.66	0.0000
Ukraine	all sample	-0.0094	0.0039	0.0231	6.3300	-0.1645	0.0563	0.0065	0.5050	41.63	0.0000
	before 1989	-0.0046	0.0020	0.0269	(0.0175)	-0.0715	0.0080	0.0000	0.3680	9.91	0.0000
	after 1989	-0.0114	0.0078	0.1560		-0.3873	0.0524	0.0000	0.6920	41.56	0.0000
Poland	all sample	-0.0031	0.0015	0.0443	4.4200	-0.0859	0.0239	0.0016	0.3170	23.14	0.0000
	before 1989	-0.0042	0.0014	0.0085	(0.0472)	-0.0676	0.0288	0.0282	0.2620	11.35	0.0000
	after 1989	0.0008	0.0019	0.6617		-0.1881	0.1236	0.1423	0.5280	15.65	0.0000
Romania	all sample	-0.0065	0.0023	0.0146	8.0600	-0.0741	0.0249	0.0117	0.7260	21.27	0.0000
	before 1989	-0.0048	0.0017	0.0176	(0.0149)	-0.0241	0.0234	0.3242	0.7990	26.39	0.0000
	after 1989	0.0013	0.0008	0.1614		-0.0924	0.0426	0.0510	0.7490	33.06	0.0000
Belarus	all sample	-0.0053	0.0034	0.1524	2.8500	-0.1516	0.0867	0.1186	0.5500	16.42	0.0000
	before 1989	-0.0101	0.0067	0.1695	(0.1299)	-0.2295	0.1360	0.1300	0.7660	12.84	0.0000
	after 1989	-0.0001	0.0018	0.9644		-0.4539	0.1107	0.0034	0.4370	8.55	0.0000
Bulgaria	all sample	-0.0016	0.0026	0.5666	0.6500	-0.0635	0.0174	0.0148	0.3450	5.77	0.0000
	before 1989	-0.0015	0.0032	0.6482	(0.4576)	-0.0470	0.0099	0.0051	0.3830	4.59	0.0000
	after 1989	0.0013	0.0037	0.7402		-0.2676	0.0599	0.0066	0.4050	2.04	0.0416
Hungary	all sample	-0.0046	0.0018	0.0515	5.7800	-0.1440	0.0433	0.0209	0.5300	12.10	0.0000
	before 1989	-0.0052	0.0029	0.1403	(0.0613)	-0.1994	0.0339	0.0020	0.2730	3.23	0.0012
	after 1989	-0.0040	0.0014	0.0353		-0.0859	0.0764	0.3121	0.7070	11.62	0.0000
Fr. Czechosl.	all sample	-0.0040	0.0021	0.1214	3.0200	-0.0874	0.0289	0.0293	0.6580	12.49	0.0000
	before 1989	-0.0068	0.0021	0.0234	(0.1430)	-0.0540	0.0347	0.1803	0.6430	8.63	0.0000
	after 1989	0.0010	0.0009	0.3523		-0.0909	0.0537	0.1514	0.5350	7.18	0.0000
Baltic States	all sample	-0.0030	0.0014	0.0888	5.2600	-0.0953	0.0188	0.0039	0.6240	13.46	0.0000
	before 1989	-0.0014	0.0011	0.2796	(0.0703)	-0.0508	0.0047	0.0001	0.2050	2.51	0.0122
	after 1989	-0.0021	0.0016	0.2510		-0.0359	0.0253	0.2162	0.3500	4.41	0.0000

std. err. are Driscoll - Kraay robust standard errors; ACSC is the average absolute value of the off-diagonal elements of the correlation matrix of the regression residuals of the fixed effects model; PCS is the Pesaran (2004) cross-section independence test; HWH is the modified Hausman (1978) test for the case when all the sample is considered; p-values are reported in round parentheses.

Next, the analysis turns to investigating the presence of a unit root taking into consideration the panel structure of the data. When using classical panel data techniques, the growth rates and the city sizes can be looked at as two different inputs and the procedure for filling some of the missing growth rates described above is employed. However, an even major problem arises when the unit root tests are considered. In this case, the input consists only in the city size data. Testing for a unit root in a time series with missing observations has received little attention in the econometric literature. Shin and Sarkar (1996) tested for a unit root in a AR(1) time-series using irregularly observed data and obtain the limiting distributions associated with the case where the gaps are ignored (i.e. the series are closed), and with the case where the gaps are replaced with the last available observation. They show that replacing the gaps with the last observation, or simply ignoring the gaps, does not alter the usual asymptotic results associated with DF statistics. Shin and Sarkar (1996) also investigated the finite sample properties of the two alternatives of dealing with missing observations in the case of an "A-B sampling scheme", where A is the number of available observations and B is the number of missing observations. Their simulation results show that the unit root test performs relatively well in small samples. Shin and Sarkar (1994) investigated a unit root test for an ARIMA(0,1,q) model with irregularly observed sample and prove to have the same asymptotic distribution as the DF statistics for the complete data situation. Some simulation results for the ARIMA(0,1,1) model show that the sizes of the tests for A-B = 6-1, 5-2 and 4-3 were similar to those for the case where there are no missing observations (i.e. A-B=7-0).

When dealing with time series data with missing observations, the other most common technique besides ignoring the gaps, and replacing the gaps with the last available observation, consists in filling the gaps with a linear interpolation method. It could be argued that instead of using the last available observation to fill these gaps, a linear interpolation between the known observations could provide a "smoother" alternative of dealing with gaps. However, the distributional implications of such a procedure require careful consideration, even in large samples. Giles (1999) extended the results of Shin and Sarkar (1996) and investigated the behavior of unit root tests when a linear interpolation method for dealing with the gaps in the data is employed. They prove that the limiting distribution includes an adjustment factor which results in critical values that are less negative than for the usual DF statistic. Giles (1999) also investigated the finite sample properties of the three alternatives for dealing with missing data. The findings obtained by Giles (1999) within a simulation experiment framework indicate that the unit root tests are more powerful when gaps are ignored, as compared with the other two alternatives of filling missing data. Following Giles (1999), when testing for a unit root in the case of cities over 100,000 inhabitants, the gaps are ignored. The results are reported in Table 5.1.3.

Table 5.1.3. *Unit root tests results for cities over 100,000 inhabitants for the period 1970-2007*

		URLLC		URIPS		URPCS		ZA			
		statistic	pvalue	statistic	pvalue	statistic	p-value			statistic	bkp.
Russia	all sample	-10.5586	0.0000	-6.5340	0.0000	2.3070	0.9890	Russia			
	before 1989	-27.8783	0.0000	-10.7660	0.0000	-0.9860	0.1620		average	-3.8670	1999
	after 1989	-12.1703	0.0000	-4.9730	0.0000	-6.6840	0.0000		max	-4.5920	2002
Ukraine	all sample	-2.5530	0.0053	0.9990	0.8410	1.0150	0.8450	Ukraine			
	before 1989	-	-	-	-	-	-		average	-4.1640	1993
	after 1989	-	-	-	-	-	-		max	-6.0970***	1985
Poland	all sample	-4.0467	0.0000	-1.6410	0.0500	-1.3670	0.0860	Poland			
	before 1989	-4.6524	0.0000	-0.6220	0.2670	-0.7110	0.2390		average	-4.2310	1987
	after 1989	-5.9089	0.0000	0.1470	0.5580	0.0350	0.5140		max	-3.5700	1990
Romania	all sample	-3.9243	0.0000	-2.2200	0.0130	-2.7190	0.0030	Romania			
	before 1989	-1.1504	0.1250	1.5330	0.9370	-2.0380	0.0210		average	-3.2650	1981
	after 1989	0.1505	0.5598	-0.8680	0.1930	1.3510	0.9120		max	-1.9660	1995
Belarus	all sample	-4.5845	0.0000	-3.5480	0.0000	-0.8670	0.1930	Belarus			
	before 1989	-4.0950	0.0000	-0.3580	0.3600	-2.0620	0.0200		average	-5.5840***	1989
	after 1989	-2.2261	0.0130	-0.0920	0.4630	1.0640	0.8560		max	-34.1120***	1999
Bulgaria	all sample	-0.8885	0.1871	-0.4400	0.3300	-1.0940	0.1370	Bulgaria			
	before 1989	-0.6097	0.2710	0.5820	0.7200	-1.0400	0.1490		average	-3.8340	1984
	after 1989	2.8549	0.9978	3.1260	0.9990	-0.6410	0.2610		max	-4.5170	1978
Hungary	all sample	-2.6283	0.0043	-5.2390	0.0000	-2.9440	0.0020	Hungary			
	before 1989	-6.7794	0.0000	-6.0500	0.0000	-3.5510	0.0000		average	-4.7470	1978
	after 1989	-2.2863	0.0111	-1.4060	0.0800	-0.7280	0.2330		max	-4.2150	1994
Fr. Czechosl.	all sample	-6.1552	0.0000	-4.6060	0.0000	-0.9050	0.1830	Fr. Czechosl			
	before 1989	-1.7602	0.0392	0.8580	0.8040	-0.2510	0.4010		average	-3.1240	1985
	after 1989	-2.8482	0.0022	-0.4750	0.3180	-1.0010	0.1580		max	-2.0380	1997
Baltic States	all sample	-1.2091	0.1133	1.0560	0.8540	1.1210	0.8690	Baltic States			
	before 1989	-0.4943	0.3105	0.9690	0.8340	-1.2810	0.1000		average	-4.2770	1982
	after 1989	-4.5589	0.0000	-2.0140	0.0220	0.6400	0.7390		max	-2.8640	1993

URLLC is the Levin et al (2002) panel unit root test; URIPS is the Im et al (2003) panel unit root test; URPCS is the Pesaran (2007) panel unit root test; the transformed t statistics are reported for the panel unit root tests; ZA is the Zivot and Andrews (1992) unit toot test wit structural breaks, bkp. indicates the year a breakpoint was detected ; *,** and *** denotes statistical significance at 10%, 5% and 1% level .

Again, in order to ensure a balanced panel, the analysis focuses on 108 cities in Russia, 31 in Ukraine, 23 in Poland, 13 in Romania, 9 in Belarus, and 6 for Bulgaria, Hungary, Former Czechoslovakia and the Baltic States. The unit root tests are not conducted unless at least 10 time observations are available, which is the case of Ukraine when the sample is split in the two sub-periods. When the tests indicate contradictory results, the priority is given to Pesaran (2007) test since it is robust to cross-sectional dependence. The results confirm, in general, the findings of the growth regressions. More specifically, the unit root tests indicate that, after 1989, the Gibrat`s law is valid in all the countries except Russia.

There is one major caveat of the regressions and of the unit root tests analyzed so far. That is the existence, after 1989, of a potential change in the deterministic component of the growth rates of the cities in the former communist block, at which the analysis is focused on in the next subsection.

5.1.3 Accounting for a potential structural break in 1989

First, the effect of a potential break on the previous results on the unit roots test is investigated. Regarding unit root tests, Perron (1989) pointed out that failure to account for an existing break leads to a bias resulting in an under-rejection of the unit root null hypothesis. To overcome this problem, Perron (1989) proposed allowing for an exogenous structural break in the standard ADF tests. Following this breakthrough, several authors including, Zivot and Andrews (1992) and Perron (1997) proposed determining the break point endogenously from the data. To account for a possible break in the series, a Zivot and Andrews (1992) unit root test was conducted. For each country, the largest city and a hypothetical city with the size equal to the average city size in the respective country were investigated. The last column in Table 5.1.3 reports the results. Zivot and Andrews (1992) structural break test is a sequential test which employs the full sample and a different dummy variable for each possible break date. The break date is selected at the time where the t-statistic of the ADF test is at a minimum, therefore, where the evidence is least favorable for the unit root hypothesis. Even accounting for a potential break, the hypothesis of a unit root, in the case of the "average" city, could not be rejected for any of the countries, except Belarus. This finding provides strong evidence in favor of Gibrat`s law.

When estimating the growth regressions in the previous subsection, the sample was split in two sub-periods to account for a possible change in the fulfillment of Gibrat`s law. However, it could be argued that splitting the data into subsets may lead to a loss in efficiency due to the reduction in the sample size. Therefore, another alternative to control for a potential change in the deterministic component of the growth rates of the cities is also employed. More specifically, a dummy variable, taking the value zero before 1989 and the value one afterwards, is introduced in the growth regressions. The results are reported in Table 5.1.4.

Table 5.1.4. *Structural breaks in the growth regressions for cities over 100,000 inhabitants for the period 1970-2007*

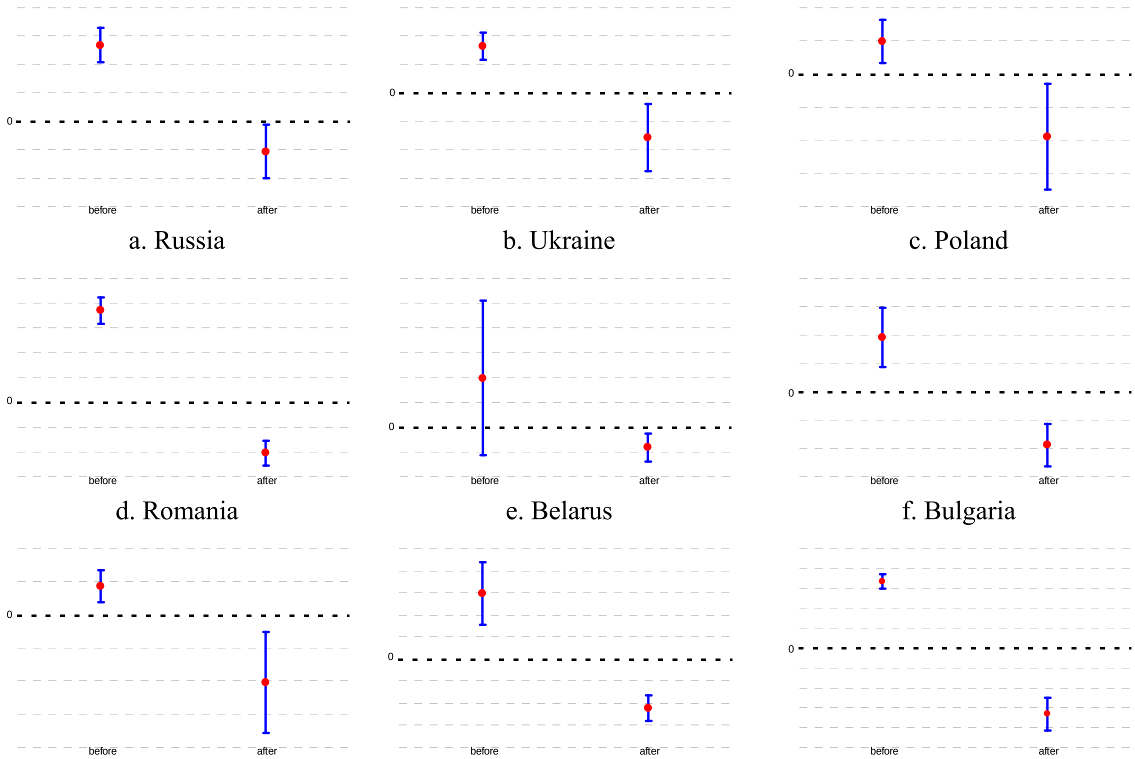
	Russia	Ukraine	Poland	Romania	Belarus	Bulgaria	Hungary	Fr. Czechosl.	Baltic States
Pooled regression									
ln(Size)	-0.0054	-0.0078	-0.0023	-0.0016	-0.0034	0.0002	-0.0045	-0.0031	-0.0017
	[0.0027]	[0.0038]	[0.0014]	[0.0012]	[0.0029]	[0.0023]	[0.0018]	[0.0018]	[0.0009]
	(0.0491)	(0.0473)	(0.1109)	(0.2316)	(0.2764)	(0.9261)	(0.0548)	(0.1460)	(0.1294)
postcom	-0.0156	-0.0256	-0.0156	-0.0302	-0.0167	-0.0210	-0.0218	-0.0152	-0.0269
	[0.0035]	[0.0084]	[0.0030]	[0.0078]	[0.0096]	[0.0042]	[0.0051]	[0.0037]	[0.0028]
	(0.0000)	(0.0047)	(0.0000)	(0.0022)	(0.1190)	(0.0040)	(0.0079)	(0.0096)	(0.0002)
Fixed effects regression									
	Russia	Ukraine	Poland	Romania	Belarus	Bulgaria	Hungary	Fr. Czechosl.	Baltic States
ln(Size)	-0.2144	-0.1549	-0.0683	-0.0296	-0.2122	-0.0332	-0.1378	-0.0631	-0.0424
	[0.0708]	[0.0605]	[0.0236]	[0.0206]	[0.1217]	[0.0199]	[0.0336]	[0.0276]	[0.0074]
	(0.0031)	(0.0157)	(0.0084)	(0.1760)	(0.1193)	(0.1552)	(0.0093)	(0.0707)	(0.0023)
postcom	0.0063	-0.0057	-0.0100	-0.0232	0.0347	-0.0152	-0.0206	-0.0096	-0.0234
	[0.0092]	[0.0090]	[0.0018]	[0.0072]	[0.0235]	[0.0051]	[0.0037]	[0.0024]	[0.0021]
	(0.4962)	(0.5324)	(0.0000)	(0.0073)	(0.1785)	(0.0307)	(0.0024)	(0.0097)	(0.0001)
ACSC	0.3850	0.5010	0.2230	0.7930	0.5470	0.3280	0.3590	0.6080	0.3240
PCS	102.1650	40.8250	13.1820	34.9980	15.8190	5.2890	8.1830	11.5450	6.1380
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

postcom is a dummy variable taking the value zero before 1989 and the value one aftewards; Driscoll - Kraay robust standard errors are reported in squared parentheses; p-values are reported in round parentheses; ACSC is the average absolute value of the off-diagonal elements of the correlation matrix of the regression residuals of the fixed effects model; PCS is the Pesaran (2004) cross-section independence test

The estimates of the pooled data model, which, as argued in the previous subsection, is given priority over the fixed effects model, indicate that the coefficients of the variable accounting for a change in the deterministic component are significantly different from zero in all the countries, except Belarus. As already mentioned, the non-parametric techniques employed in this paper (Li and Racine 2003; Hayfield and Racine, 2008) are appropriate for a mix of continuous and discrete data. This is convenient because it allows investigating, by means of non-parametric regression, whether the influence of discrete variables accounting for potential structural breaks is significant.

The graphs in Figure 5.1.1 depict the impact on city growth rates of the dummy variable accounting for a structural break in 1989. As it is standard in non-parametric analysis, to capture the sole influence of one variable (in this case the dummy), the other variable (in this case the relative city size) is held at the median value. The 95% distribution free (bootstrapped) error bounds, computed using 500 random samples with replacement, are also depicted. The results confirm the findings of the parametric analysis with a shift in the deterministic component detected in all the countries except Belarus.

Figure 5.1.1. *The non-parametrical estimates of the potential shift in the deterministic component of growth rates*



a. Russia   b. Ukraine   c. Poland

d. Romania   e. Belarus   f. Bulgaria

g. Hungary   h. Former Czechoslovakia   i. Baltic States

After the influence of the change in the deterministic component is accounted for, the null hypothesis of the validity of Gibrat`s law can not be rejected at any standard level of significance for six of the analyzed countries or groups of countries, respectively Poland, Romania, Belarus, Bulgaria, Former Czechoslovakia, and the Baltic States. For Hungary can not be rejected at 5%, and for Russia and Ukraine cannot be rejected at 1%.

5.1.4 Gibrat`s law using five years averages

Another caveat of the analysis using yearly data on cities over 100,000 inhabitants is given by the existence of missing data in some of the years in the time span. As argued in the previous subsections, the treatment of missing data in this study is reasonable and the consistency of econometric methods assured. However, in order to check the robustness of the results, in this subsection the analysis is also conducted using five years averages. For the last period, 2005-2007, only three years are available and, therefore, three years averages are employed.

Table 5.1.5. *Growth regressions results for cities over 100,000 inhabitants using five years averages for the period 1970-2007*

		Pooled regression			Pooled regression with dummy			
		all sample	before 1989	after 1989	all sample			
Russia	ln(Size)	-0.0023	-0.0020	-0.0006	ln(Size)	-0.0012	postcom	-0.0175
		[0.0008]	[0.0003]	[0.0004]		[0.0005]		[0.0034]
		(0.0034)	(0.0000)	(0.1618)		(0.0097)		(0.0000)
Ukraine	ln(Size)	-0.0072	-0.0057	-0.0050	ln(Size)	-0.0053	postcom	-0.0212
		[0.0019]	[0.0010]	[0.0023]		[0.0013]		[0.0047]
		(0.0004)	(0.0000)	(0.0387)		(0.0002)		(0.0001)
Poland	ln(Size)	-0.0032	-0.0058	0.0013	ln(Size)	-0.0019	postcom	-0.0166
		[0.0025]	[0.0011]	[0.0013]		[0.0022]		[0.0041]
		(0.2127)	(0.0000)	(0.3107)		(0.3946)		(0.0005)
Romania	ln(Size)	-0.0087	-0.0090	0.0010	ln(Size)	-0.0036	postcom	-0.0314
		[0.0041]	[0.0008]	[0.0008]		[0.0026]		[0.0087]
		(0.0536)	(0.0000)	(0.2599)		(0.1817)		(0.0029)
Belarus	ln(Size)	-0.0032	0.0000	0.0026	ln(Size)	0.0015	postcom	-0.0287
		[0.0018]	[0.0031]	[0.0015]		[0.0010]		[0.0031]
		(0.1159)	(0.9969)	(0.1282)		(0.1467)		(0.0000)
Bulgaria	ln(Size)	-0.0001	0.0001	0.0019	ln(Size)	0.0021	postcom	-0.0159
		[0.0013]	[0.0011]	[0.0034]		[0.0018]		[0.0037]
		(0.9265)	(0.9256)	(0.5956)		(0.2939)		(0.0049)
Hungary	ln(Size)	-0.0002	-0.0035	0.0006	ln(Size)	-0.0008	postcom	-0.0133
		[0.0008]	[0.0008]	[0.0002]		[0.0010]		[0.0036]
		(0.8148)	(0.0088)	(0.0242)		(0.4320)		(0.0099)
Fr. Czechosl.	ln(Size)	-0.0039	-0.0068	0.0005	ln(Size)	-0.0028	postcom	-0.0148
		[0.0023]	[0.0011]	[0.0005]		[0.0020]		[0.0039]
		(0.1525)	(0.0017)	(0.3546)		(0.2157)		(0.0125)
Baltic States	ln(Size)	-0.0028	-0.0011	-0.0014	ln(Size)	-0.0013	postcom	-0.0218
		[0.0008]	[0.0004]	[0.0014]		[0.0008]		[0.0040]
		(0.0121)	(0.0389)	(0.3406)		(0.1426)		(0.0015)

postcom is a dummy variable taking the value zero before 1989 and the value one afterwards; Driscoll - Kraay robust standard errors are reported in squared parentheses; p-values are reported in round parentheses.

To ensure that the panels are balanced some of the cities with missing observations were drooped. Therefore, the number of analyzed cities is 130 for Russia, 37 for Ukraine, 25 for Poland, 15 for Romania, 9 for Belarus, 7 for Bulgaria, Hungary and the Baltic States, and 6 for Former Czechoslovakia. Because the time dimension is too low (8 periods) to use panel unit root tests, only growth regression are estimated using pooled data. The results quantifying the influence of the five year average size on the annualized growth rate are reported in Table 5.1.5.

The results of the pooled regression indicates that, in the post-communist period, Gibrat`s law is valid in all of the countries, with less evidence in the case of Ukraine and Hungary. When all the sample is considered Gibrat`s law is rejected in Russia and Ukraine. However, this is contrary to the findings of the non-parametrical regressions, reported in Figure A.5.1.3 in the Appendix, that indicate the acceptance of the proportional effect law in Russia and Ukraine in all of the three subsamples.

Also in the case of using five years averages, the estimates from the parametric method, as well as the results of the non-parametric method (Figure A.5.1.4 in the Appendix), indicate that the dummy variable accounting for a change in the deterministic component has a significant influence in all the countries. After accounting for the shift in the deterministic component, the null hypothesis of the validity of Gibrat`s law can not be rejected at any standard level of significance for seven of the analyzed countries or groups of countries, respectively Poland, Romania, Belarus, Hungary, Bulgaria, Former Czechoslovakia, and the Baltic States. On the other hand, there is strong evidence against Gibrat`s law in the case of Russia and Ukraine.

5.2 Results concerning the Pareto exponent of the city size distribution

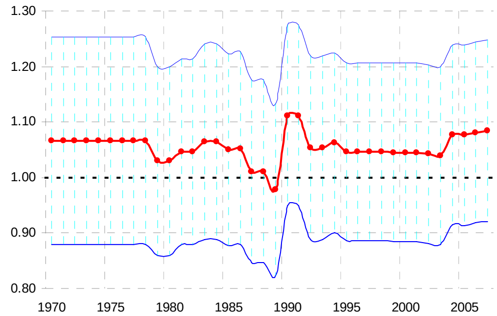
In this section, we estimate the Pareto exponent of the city size distribution for the case of CEE and CIS transition economies using data for cities over 100,000 inhabitants. In this version of the paper we employed city data on 15 countries, respectively Belarus, Bulgaria, Poland, Romania, Russian Federation, Ukraine, Estonia, Latvia, Lithuania, Bosnia and Herzegovina, Croatia, Macedonia, Serbia, Slovenia, Czech Republic, and Slovak Republic. As one can easily observe from Table 3.1 in some countries the sample size for cities over 100,000 is insufficient for estimating the Pareto coefficient. Therefore, in order to be able to perform the estimation, these counties were pooled into three groups. The first group consists of the Baltic States (Estonia, Latvia, Lithuania), the second one of the countries from the Former Yugoslavia (Bosnia and Herzegovina, Croatia, Macedonia, Serbia, Slovenia), and the last one of the countries from the Former Czechoslovakia (Czech Republic, Slovak Republic). Using the grouping procedure we estimated for each year between 1970 and 2007 the Pareto coefficient as described in section 4.2 for the remaining 10 units. The average sample sizes cities over 100,000 inhabitants for these units are as follows: Russian Federation 150, Ukraine 45, Poland 37, Romania 21, Former Yugoslavia 18, Belarus 12, Baltic States 8, Hungary 8, Former Czechoslovakia 8, and Bulgaria 8. The full results of the two estimating techniques are presented in Table A.5.2.1 in the Appendix. Table 5.2.1 summarizes the results, by presenting the average value over 1970-2007 of the two series of estimates, the standard deviation, the minimum and the maximum value over the period.

Table 5.2.1. *Regression and MLE estimates for the Pareto coefficient*

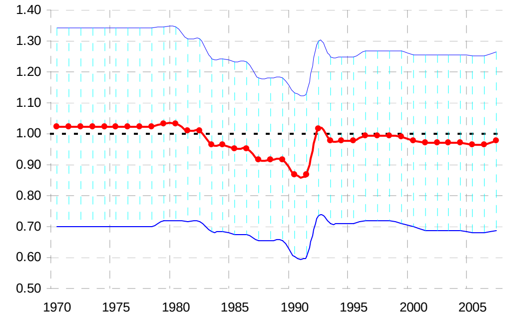
	Regression estimates				MLE estimates			
	Average	Std. dev	Min	Max	Average	Std. dev	Min	Max
Russian Federation	1.2600	0.0480	1.1360	1.3250	1.0080	0.1700	0.3790	1.1110
Ukraine	1.1980	0.0290	1.1650	1.2440	0.9810	0.0420	0.8680	1.0320
Poland	1.4320	0.0230	1.3410	1.4560	1.3240	0.0730	1.1700	1.4040
Romania	1.4050	0.0560	1.2750	1.4760	1.4710	0.1780	1.2080	2.0660
Former Yugoslavia	1.3310	0.0790	1.2540	1.5880	1.5230	0.1310	1.2430	1.7740
Belarus	1.2450	0.0910	1.1510	1.3990	1.2790	0.1320	1.1040	1.4790
Baltic States	1.0990	0.0270	1.0620	1.1440	1.1640	0.1010	0.9880	1.4010
Hungary	0.8940	0.0730	0.7430	0.9740	1.5360	0.1510	1.2330	1.7800
Former Czechoslovakia	1.1080	0.0550	1.0510	1.2350	1.1710	0.1770	0.9090	1.4330
Bulgaria	1.1640	0.1020	0.7600	1.2510	1.4270	0.1040	1.2290	1.5500

For all the countries in the dataset the regression technique give more stable estimates, since the standard deviation of the regression estimates series is lower than the one of the MLE estimates. Figure 5.2.1 depicts the estimated Pareto exponents, using MLE, and their corresponding 95% confidence bands. The similar results for the regression estimates are depicted in Figure A.5.2.1 in the Appendix. The dynamics of the difference between the two estimates series is presented in Figure A.5.2.2 in the Appendix.

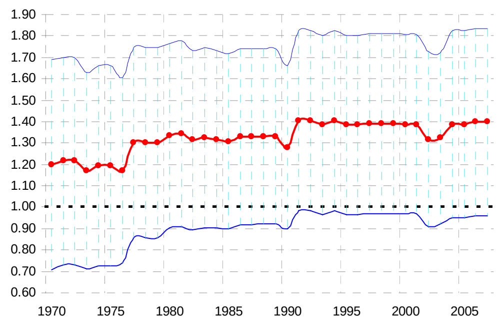
Table 5.2.1. *The dynamics of the MLE estimate of the Pareto exponent*



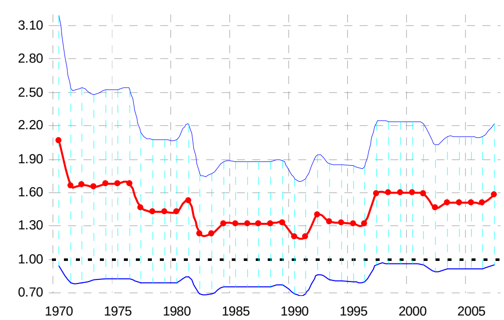
a. Russian Federation



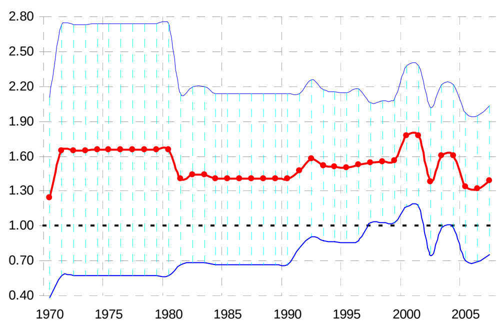
b. Ukraine



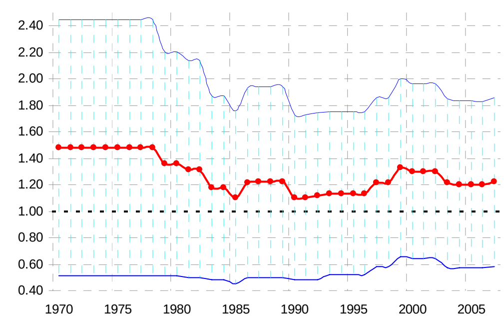
c. Poland



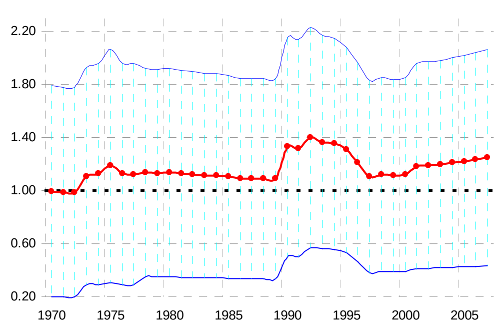
d. Romania



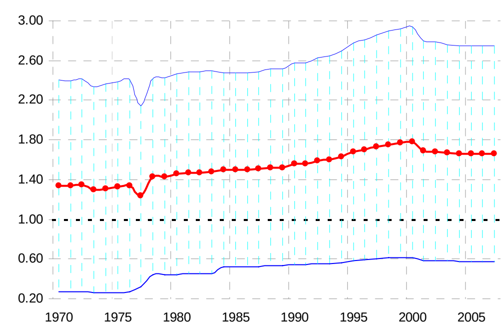
e. Former Yugoslavia



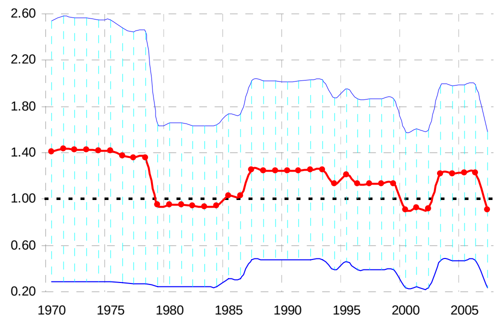
f. Belarus



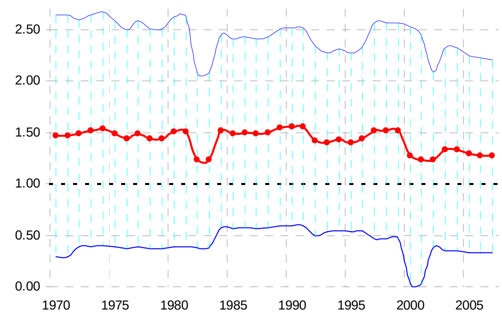
g. Baltic States



h. Hungary



i. Former Czechoslovakia



j. Bulgaria

For the large majority of countries and time periods the estimated coefficient is higher than one. However, as it is easily observable from Figure 5.2.1, one can not reject that the Pareto exponent is significantly different from one, and therefore it seems that the Zipf Law holds. This is in line with other studies in the literature that obtained favorable evidence of Zipf's Law in the upper-tail distribution of cities. On the other hand, we have to be skeptical of the results since we employed asymptotic standard errors and the sample sizes for some of the countries are rather reduced. The analysis can be improved by computing standard errors using bootstrapping techniques, which are expected to provide more robust results. Also, it is essential to obtain better standard errors since they are employed in quantifying the consensus estimate of the Pareto exponent.

In the next section we employ detailed data to determine the distribution of city size using different concurrent parametric models. Levy (2009) points out that, while the lognormal distribution fits the empirical data extremely well for 99.4 percent of the size range, as argued by Eeckhout (2004), in the top 0.6 percent range of the largest cities, the size distribution diverges dramatically and systematically from the lognormal distribution, and instead is much better described by a power law. Also, as pointed out by Eeckhout (2009), a log-normal distribution of the tails does not mean that a Pareto fit does not exist.

5.3 Results concerning the non-Pareto behavior of the city size distribution

In this section of the paper we present wide-scale comparisons of the estimates of city size distribution obtained using power laws, the Weber-Fechner Law, and the logarithmic hierarchy model as described in section 4.3.

We consider the development of cities in Kazakhstan, Uzbekistan, Kyrgyzstan, Tajikistan and Turkmenistan that represent the so-called Central Asia region of the CIS [1]. Armenia, Azerbaijan and Georgia representing the so-called CIS Caucasus.

Since Zipf's Law with $\alpha \approx 1$ holds only for the tails of distributions of cities that include only large cities plus one or more mega-cities which contrast sharply in size to the other cities, we will examine the occurrence of the Weber-Fechner Law in relation to the size of cities and their rank. While Zipf's Law corresponds to a log-log relationship between the ranks of large cities and their sizes

$$\log Rank = c - \alpha \log Size$$

with the regression coefficient α equal to 1, the Weber-Fechner Law has the form

$$\log Size = \beta - \gamma \cdot Rank \ .$$

That is, in the case of the Weber-Fechner law, the rank of the city changes in arithmetic progression with the change of the size of the city in geometric progression. In this context one of our research objectives is to compare Weber's constants γ for the distribution of cities in different countries. Such comparisons will be further used to describe the differences of urbanization processes in different countries and the impact of administrative measures aimed at restricting the size of the capitals and large cities in post-Soviet countries like Russia, Belarus, Central Asian countries and Caucasus countries. This analysis is essential for any attempts to forecast the development of urbanization in different countries.

While Zipf's Law is inherent to the communities, the Weber-Fechner Law is typical for living organisms. The Weber-Fechner Law says: «The perception will grow in arithmetic progression, when stimuli grow in geometric progression». This Law was published in G. Fehner's book "Elements of Psychophysics" in 1859. The Law was discovered in the early 19th century by E. Weber a German physiologist and psychologist. He studied in detail the link between perception and stimuli when he determined how to

[1] At the summit of Central Asian states held in 1992, the President of Kazakhstan Nursultan Nazarbayev proposed to give up the term "Central Asia and Kazakhstan" in favor of the concept of "Central Asia" that covers all post-Soviet states in the region.

change a stimulus for this change to be noticed by a person. It turned out that a ratio of stimulus change (intense) to its initial value is constant:

$$\frac{\Delta I}{I} = k \ ,$$

where I is the stimulus measure, ΔI is the stimulus change/intense, and k is Weber's constant.

Let i=1, …, n, be the rank of cities and towns in consideration. Let us interpret the rank of cities/towns as a measure of perception that changes on an arithmetic progression with a step (a difference) equal to 1. Let us also interpret the size of a city/town N_i (the number of inhabitants) as the measure of a stimulus, since ranking has been made according to this parameter. Denote by $\Delta N_i = N_i - N_{i-1}$, i=2,…, n, the change in the stimulus. Let us suppose that

$$\frac{\Delta N_i}{N_i} = k = \text{const.}$$

Changing ΔN_i by differential dN_i, we have

$$\frac{dN_i}{N_i} = d\ln N_i = k = \text{const.}$$

Solving the above differential equation, we obtain

$$\ln N_i = c + k \cdot i,$$

where c and k are some constants. Hence,

$$N_i = Aq^i,$$

where $A=e^c$, $q=e^k$. In the sequel we will interpret q as the denominator of geometric progression, that corresponds to the change in the "stimulus" N_i .

5.3.1. Zipf's Law

The following are the estimation results for the log-log rank-size regression with the optimal shift 1/2 for Russia, Belarus, Central Asian and Caucasus cities. That is, the estimated regression is

$$\ln(i-1/2) = a - \zeta \cdot \ln N_i,$$

where $N_1 \geq N_2 \geq \cdots \geq N_m$ are the ordered city sizes in the samples considered and i denotes the rank of i-th city.

Russia

Based on the number of urban dwellers in Rusia in 1897-2009, we estimated regression coefficients $\ln(i\text{-}1/2) = a - \zeta \cdot \ln N_i$, where N_i - size city (population size), i - rank of the cities. The results are presented in Table A.5.3.1 in the Appendix.

Figure 5.3.1. *Russian cities in 1897-2009 (the log-log scale)*

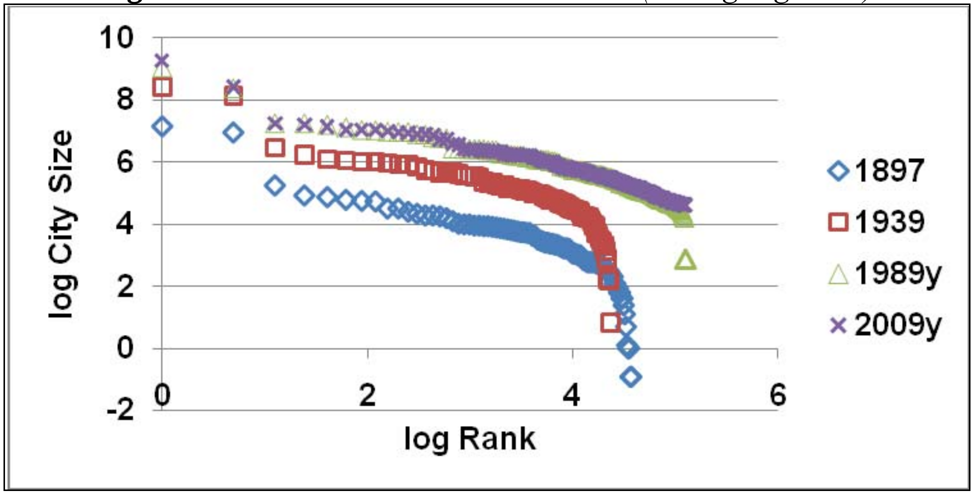


Table 5.3.1. 95% confidence interval for coefficient ζ of largest cities in Russia (with the population above 100 thousand people)

Years	**Sample size *n***	**Estimated coefficient ζ**	**Standard error of the estimation.**[2]	**95% confidence interval for the coefficient ζ**
1897	8	0.82414	0.41207	(0.016, 1.632)
1926	20	1.11359	0.352148	(0.423, 1.804)
1939	51	1.27545	0.252577	(0.780, 1.771)
1959	66	1.3038	0.226962	(0.859, 1.749)
1970	75	1.29735	0.211856	(0.882, 1.713)
1979	138	1.26617	0.152429	(0.967, 1.565)
1989	151	1.23767	0.14244	(0.958, 1.517)

[2] Standard error of coefficient ζ is calculated according to the formula $\sqrt{\frac{2}{n}}\zeta$.

2002	159	1.22786	0.13771	(0.958, 1.498)
2003	159	1.22668	0.137578	(0.957, 1.496)
2004	159	1.22984	0.137932	(0.959, 1.500)
2005	163	1.23317	0.136598	(0.965, 1.501)
2006	163	1.23459	0.136755	(0.967, 1.503)
2007	163	1.23494	0.136794	(0.967, 1.503)
2008	163	1.23463	0.13676	(0.967, 1.503)
2009	164	1.23284	0.136144	(0.966, 1.500)

According to the estimation results in Table 5.3.1, the confidence intervals for all the samples considered contain the threshold value ζ=1 that corresponds to the Zipf's law. Conclusion: Zipf's Law holds for the cities of the Russia.

Belarus

Based on the number of urban dwellers in Belarus in 1970-2009, we estimated regression coefficients. The results are presented in Table 5.3.2.

Table 5.3.2. 95% confidence interval for coefficient ζ of cities in Belarus

Years	**Sample size**	**Truncation, %**	***n***	**Estimated coefficient** ζ	**Standard error of the estimation** S.e.=$\sqrt{\frac{2}{n}}\hat{\zeta}$	**95% confidence interval for** ζ
1970	198	20	41	1.038122	0.229	(0.589, 1.488)
		10	21	0.879841	0.272	(0.348, 1.412)
1979	200	20	41	1.056392	0.233	(0.599, 1.514)
		10	21	0.882730	0.272	(0.349, 1.417)
1989	202	20	41	1.050578	0.232	(0.596, 1.505)
		10	21	0.872941	0.269	(0.345, 1.401)
1990	202	20	41	1.044586	0.231	(0.592, 1.497)
		10	21	0.870052	0.269	(0.344, 1.396)
1991	202	20	41	1.036595	0.229	(0.588, 1.485)
		10	21	0.858082	0.265	(0.339, 1.377)
1992	202	20	41	1.040826	0.230	(0.590, 1.491)
		10	21	0.865029	0.267	(0.342, 1.388)
1993	202	20	41	1.037825	0.229	(0.589, 1.487)
		10	21	0.861267	0.266	(0.340, 1.382)
1994	202	20	41	1.033694	0.228	(0.586, 1.481)
		10	21	0.856657	0.264	(0.338, 1.375)

1995	202	20	41	1.031961	0.228	(0.585, 1.479)
		10	21	0.852876	0.263	(0.337, 1.369)
1997	203	20	41	1.022432	0.226	(0.580, 1.465)
		10	21	0.852783	0.263	(0.337, 1.369)
1998	205	20	41	1.036009	0.229	(0.588, 1.484)
		10	21	0.855515	0.264	(0.338, 1.373)
1999	205	20	41	1.034578	0.229	(0.587, 1.482)
		10	21	0.844820	0.261	(0.334, 1.356)
2000	205	20	41	1.034670	0.229	(0.587, 1.483)
		10	21	0.845432	0.261	(0.334, 1.357)
2001	207	20	41	1.035923	0.229	(0.587, 1.484)
		10	21	0.848378	0.262	(0.335, 1.362)
2002	207	20	41	1.038229	0.229	(0.589, 1.488)
		10	21	0.851213	0.263	(0.336, 1.366)
2003	206	20	41	1.041158	0.230	(0.590, 1.492)
		10	21	0.854345	0.264	(0.338, 1.371)
2004	206	20	41	1.044055	0.231	(0.592, 1.496)
		10	21	0.857012	0.264	(0.339, 1.375)
2005	206	20	41	1.047367	0.231	(0.594, 1.501)
		10	21	0.861484	0.266	(0.340, 1.383)
2006	206	20	41	1.049916	0.232	(0.595, 1.504)
		10	21	0.864966	0.267	(0.342, 1.388)
2007	207	20	41	1.052351	0.232	(0.597, 1.508)
		10	21	0.867555	0.268	(0.343, 1.392)
2008	206	20	41	1.056118	0.233	(0.599, 1.513)
		10	21	0.871507	0.269	(0.344, 1.399)
2009	206	20	41	1.059402	0.234	(0.601, 1.518)
		10	21	0.874751	0.270	(0.346, 1.404)

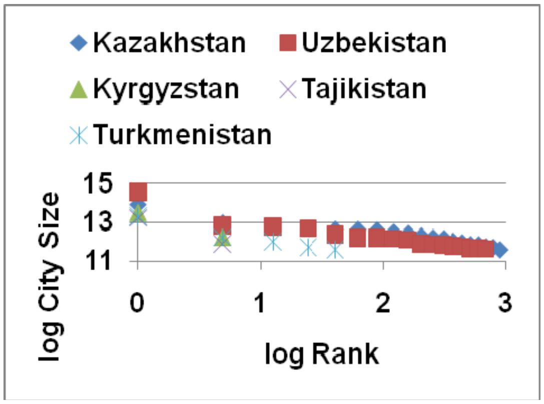
According to the estimation results in Table 5.3.2, the confidence intervals for all the samples considered contain the threshold value ζ=1 that corresponds to the Zipf's law. Conclusion: Zipf's Law holds for the cities of the Belarus.

Central Asia

Based on the number of urban dwellers in Central Asian countries in 1999, we estimated regression coefficients[3]. The results are presented in Table 5.3.3.

Figure 5.3.2. *Central Asian cities in 1999 (the log-log scale).*

[3] In 1999, the intersection data of all the countries of Central Asia.



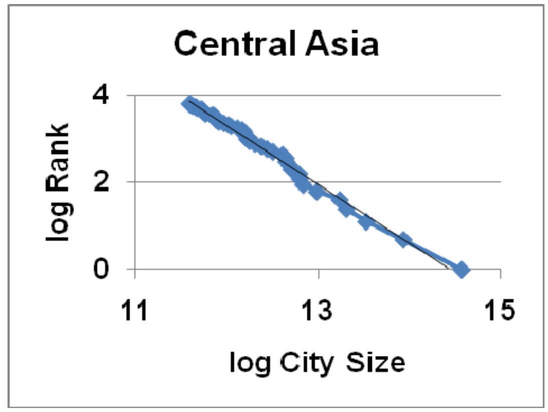


Table 5.3.3. Estimates of the tail index ζ (City sizes greater than 100 thousand people, data for 1999)

Country	Number of cities, m	Estimated tail index, ζ	Standard errors, $\sqrt{(2/m)}\ \zeta$	95% confidence intervals for the tail index ζ
Kazakhstan	19	1.646905	0.534327	(0.600, 2.694)
Uzbekistan	17	1.266066	0.434257	(0.415, 2.117)
Kyrgyzstan	2	0.857973	0.857973	(-0.824, 2.540)
Tajikistan	2	0.827545	0.827545	(-0.794, 2.450)
Turkmenistan	5	1.258920	0.796211	(-0.302, 2.819)
Central Asia	45	1.491596	0.314456	(0.875, 2.108)

According to the estimation results in Table 5.3.3, the confidence intervals for all the samples considered contain the threshold value ζ=1 that corresponds to the Zipf's law.

Conclusion: Zipf's Law holds for the cities of the Central Asia.

Caucasus

Based on the number of urban dwellers in countries of the Caucasus in 2007, we estimated regression coefficients[4]. The results are presented in Table 5.3.4.

[4] In 2007, the intersection data of all the countries of Caucasus.

Figure 5.3.3. *Cities of the Caucasus in 2007 (the log - log scale)*

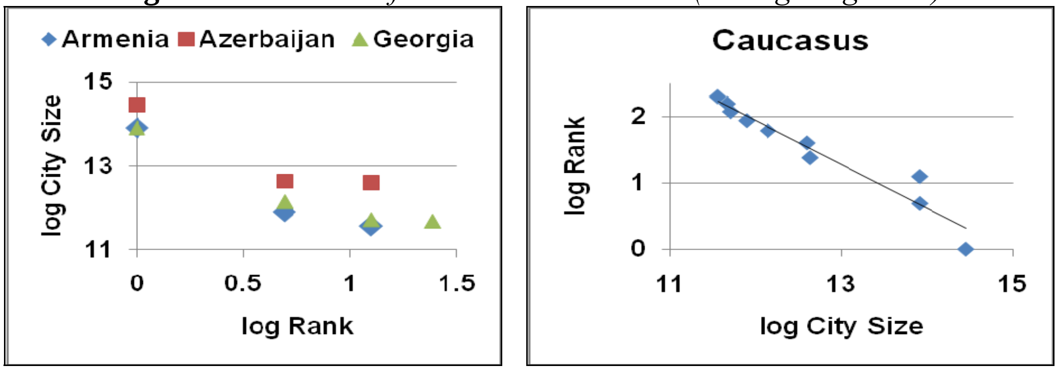


Table 5.3.4. Estimates of the tail index ζ
(City sizes greater than 100 thousand people, data for 2007*)*

Country	Number of cities, *m*	Estimated tail index, $\hat{\zeta}$	Standard errors, $\sqrt{(2/m)}\,\hat{\zeta}$	95% confidence intervals for the tail index ζ
Armenia	3	0.635413	0.518813	(-0.381, 1.652)
Azerbaijan	3	0.740743	0.604814	(-0.445, 1.926)
Georgia	4	0.780854	0.552147	(-0.301, 1.863)
Caucasus	10	0.813744	0.363917	(0.100, 1.527)

According to the estimation results in Table 5.3.4, the confidence intervals for all the samples considered contain the threshold value ζ=1 that corresponds to the Zipf's law. Conclusion: Zipf's Law holds for the cities of the countries of the Caucasus.

5.3.2. Weber-Fechner Law

Russia

Estimates of the coefficients of regression $\ln N_i = c + k \cdot i$ based on the data on the population of the Russian cities for the years 1897-2009 as well as the coefficients of the equation $N_i = Aq^i$ are given in table A5.3.2 in the Appendix and Table 5.3.5.

Table 5.3.5. Parameters of regression of logarithms of the population N_i for cities of Russia agaist its ranks: $\ln N_i = c + k \cdot i$, $N_i = Aq^i$, where $A = e^c$, $q = e^k$ (except for Moscow and Saint-Petersburg)

Years	*t*	*c*	*k*	*A*	*q*	*r=1/q*
1897	7	5.078591	-0.04017	160.5477	0.960631	1.040983
1926	36	5.819091	-0.04973	336.6659	0.951491	1.050982
1939	49	6.46233	-0.04125	640.5518	0.959585	1.042117
1959	69	6.801082	-0.03611	898.8193	0.964534	1.03677
1970	80	6.959886	-0.0303	1053.513	0.970157	1.030761
1979	89	6.71537	-0.01641	824.989	0.983729	1.01654
1989	99	6.823465	-0.01585	919.1644	0.984278	1.015973
2002	112	6.761217	-0.01502	863.6927	0.98509	1.015135
2003	113	6.757072	-0.01496	860.1201	0.985147	1.015077
2004	114	6.755904	-0.01497	859.116	0.985139	1.015086
2005	115	6.736914	-0.01457	842.9554	0.985534	1.014679
2006	116	6.736782	-0.01455	842.8441	0.985552	1.014659
2007	117	6.733465	-0.01455	840.053	0.98556	1.014651
2008	118	6.733453	-0.01455	840.0429	0.985558	1.014653
2009	119	6.734632	-0.01455	841.0339	0.98556	1.014651

In summary, the following conclusions can be made:

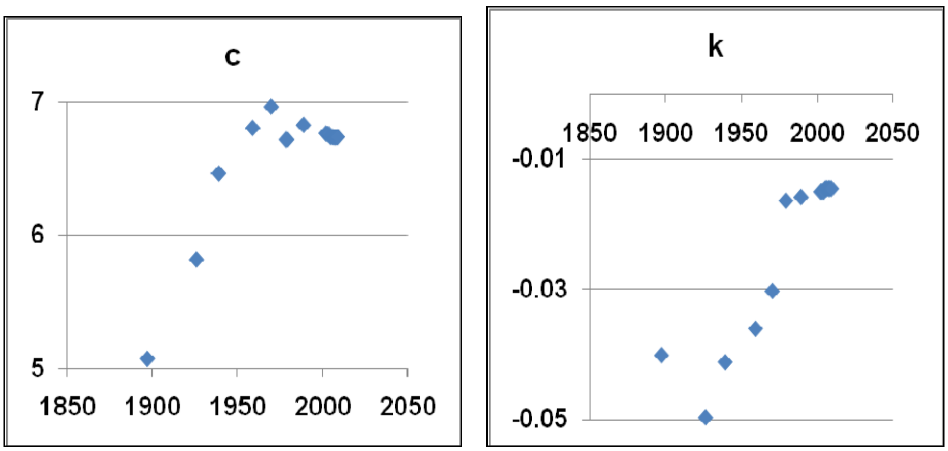
1. Development of cities of Russia can be well explained by the Weber-Fechner Law (see table A5.3.2 in the Appendix).
2. Weber constant from the year 2006 has been equal to 0.01455.
3. For the change in the population to be noticeable (for infrastructure, administrative decisions) this change should be greater than 1.5% of the population of the city (r=1/q=1.015). Therefore, the decisions (administrative, economic, ecological etc.) should be changed if the population of the cities increases by more than 1.5%.

4. Moscow and Saint-Petersburg have the special status and do not comply with the Weber-Fechner Law. Therefore, while forecasting the results of urbanization Moscow and Saint-Petersburg should be given in the separate column, that is independent on the decisions, adopted for other cities.

Change in the Weber coefficients

Curves regresionnyh dependencies ln Ni = $c+k\cdot i$, the corresponding parameters from Table 5.3.5, are shown in Figure 5.3.4.

Figure 5.3.4. *Regressions ln Ni = c+k·i for Russian cities in 1897-2009*



In Table A5.3.3 in the Appendix provides the estimation results for the regression of the (estimated) parameters ***c*** and ***k*** on the time trend (the ranks *t* of years 1897, 1898, ..., 2009 and the dummy political variables ***P*1**, ***P*2, *P*3** (***P*1** takes the value 0 before the Great October Revolution and the value 1 after the revolution, ***P*2** takes the value 0 before the Second World War and a value of 1 after the Second World War, ***P*3** takes the value 0 to the collapse of the USSR and the value 1 after the collapse of the USSR).

Acceptable from the standpoint of the Statistical significance of regression coefficients and the model as a whole, the model interaction are 2, 3 and 4 for the parameter *c* (all coefficients are significant with a probability of error less than 0.09). For the parameter *k* as all coefficients are significant with a probability of error of no more than 0.09.

Thus, there is

1. $c = 5.078591 + 1.06212 \cdot P1 + 0.63006 \cdot P2$,

2. $c = -31.47273 + 0.019399 \cdot t - 0.687989 \cdot P3$,

3. $c = 0.002678 \cdot t + 0.966226 \cdot P1 + 0.57245 \cdot P2 - 0.164946 \cdot P3$,

$k = -1.349853 + 0.00069 \cdot t - 0.029829 \cdot P1 - 0.008002 \cdot P2 - 0.011622 \cdot P3$,

1. $$N_i = \exp(5.078591 + 1.06212 \cdot P1 + 0.63006 \cdot P2) \cdot \\ \cdot \exp((-1.349853 + 0.00069 \cdot t - 0.029829 \cdot P1 - 0.008002 \cdot P2 - 0.011622 \cdot P3) \cdot i)$$

2. $$N_i = \exp(-31.47273 + 0.019399 \cdot t - 0.687989 \cdot P3) \cdot \\ \cdot \exp((-1.349853 + 0.00069 \cdot t - 0.029829 \cdot P1 - 0.008002 \cdot P2 - 0.011622 \cdot P3) \cdot i)$$

3. $$N_i = \exp(0.002678 \cdot t + 0.966226 \cdot P1 + 0.57245 \cdot P2 - 0.164946 \cdot P3) \cdot \\ \cdot \exp((-1.349853 + 0.00069 \cdot t - 0.029829 \cdot P1 - 0.008002 \cdot P2 - 0.011622 \cdot P3) \cdot i)$$

that is $N_i = A(t,P) \cdot q(t,P)^i$, where

1. $A(t,P) = \exp(5.078591 + 1.06212 \cdot P1 + 0.63006 \cdot P2)$,
2. $A(t,P) = \exp(-31.47273 + 0.019399 \cdot t - 0.687989 \cdot P3)$
3. $A(t,P) = \exp(0.002678 \cdot t + 0.966226 \cdot P1 + 0.57245 \cdot P2 - 0.164946 \cdot P3)$

$q(t,P) = \exp(-1.349853 + 0.00069 \cdot t - 0.029829 \cdot P1 - 0.008002 \cdot P2 - 0.011622 \cdot P3)$.

Thus, the Great October Revolution and the Second World War gave the effect of increasing the size of the largest cities of Russia and the Soviet collapse gave the effect of reducing the size (ceteris paribus)

Belarus

Estimates of the coefficients of regression $\ln N_i = c + k \cdot i$ based on the data on the population of the Belarusian cities for the years 1970-2009 as well as the coefficients of the equation $N_i = Aq^i$ are given in tables A5.3.4, A.5.3.5 in the Appendix and Table 5.3.6.

Table 5.3.6. Parameters of regression of logarithms of the population N_i for cities of Belarus agaist its ranks: ln $N_i = c + k \cdot i$, $N_i = Aq^i$, where $A = e^c$, $q = e^k$.

Year	***c***	***k***	***A***	***q***	***r=1/q***
1970	3.656932	-0.018014	38.742	0.9821	1.0182
1979	4.017004	-0.019605	55.534	0.9806	1.0198
1989	4.333892	-0.02099	76.240	0.9792	1.0212
1990	4.366013	-0.021041	78.729	0.9792	1.0213
1991	4.382248	-0.021133	80.018	0.9791	1.0214
1992	4.398832	-0.021238	81.356	0.9790	1.0215
1993	4.416907	-0.021356	82.840	0.9789	1.0216
1994	4.432715	-0.021437	84.160	0.9788	1.0217
1995	4.437681	-0.021431	84.579	0.9788	1.0217
1997	4.457189	-0.021565	86.245	0.9787	1.0218
1998	4.43477	-0.021255	84.333	0.9790	1.0215
1999	4.423677	-0.021485	83.402	0.9787	1.0217
2000	4.428232	-0.021539	83.783	0.9787	1.0218
2001	4.414205	-0.021282	82.616	0.9789	1.0215
2002	4.416414	-0.021354	82.799	0.9789	1.0216
2003	4.41247	-0.021361	82.473	0.9789	1.0216
2004	4.409145	-0.021384	82.199	0.9788	1.0216
2005	4.410506	-0.021489	82.311	0.9787	1.0217
2006	4.41131	-0.021573	82.377	0.9787	1.0218
2007	4.414477	-0.02166	82.639	0.9786	1.0219
2008	4.416968	-0.021725	82.845	0.9785	1.0220
2009	4.421704	-0.021776	83.238	0.9785	1.0220

In summary, the following conclusions can be made:

1. Development of cities of Belarus can be well explained by the Weber-Fechner Law (see table A5.3.4 in the Appendix).
2. Weber constant from the year 2006 has been equal to 0.022.
3. For the change in the population to be noticeable (for infrastructure, administrative decisions) this change should be greater than 2.2% of the population of the city (r=1/q=1.022). Therefore, the decisions (administrative, economic, ecological etc.) should be changed if the population of the cities increases by more than 2.2%.

Change in the Weber coefficients

Curves regresionnyh dependencies ln $Ni = c+k \cdot i$,, the corresponding parameters from Table 5.3.6, are shown in Figures 5.3.5, 5.3.6.

Figure 5.3.5. Change of parameters *a* of the Weber-Fechner Model *Rank*=*c*+*k*ln*Size* with 1970 for 2009 for settlements of Belarus

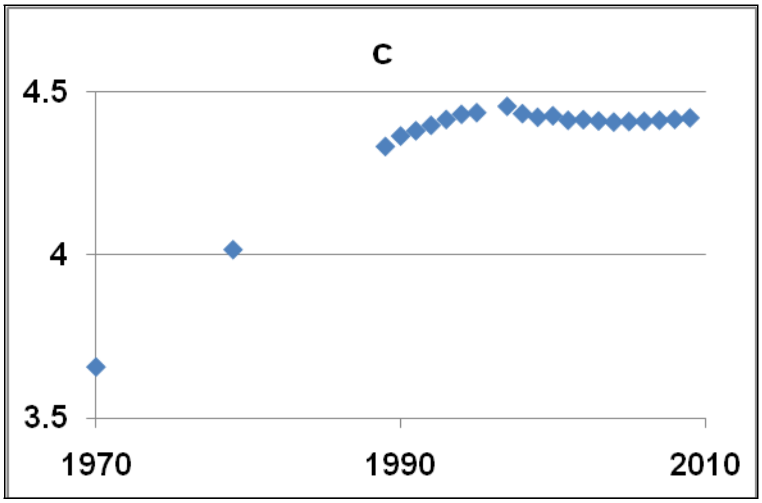
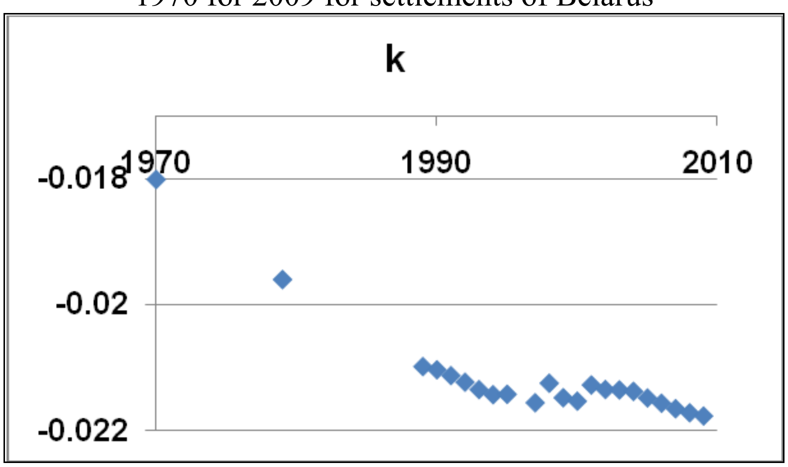


Figure 5.3.6. Change of parameters *k* of the Weber-Fechner Model *Rank*=*c*+*k*ln*Size* with 1970 for 2009 for settlements of Belarus



Calculations show that the collapse of the Soviet Union at the rate of urban growth in the Belarus statistically significant effects are not influence.

Central Asia

Estimates of the coefficients of regression $\ln N_i = c + k \cdot i$ based on the data on the population of the Central Asia cities for the year 1999 as well as the coefficients of the equation $N_i = Aq^i$ are given in tables A5.3.6 in the Appendix and Table 5.3.7.

Table 5.3.7. Parameters of regression of logarithms of the population N_i for cities of Central Asia in 1999 agaist its ranks: $\ln N_i = c + k \cdot i$, $N_i = Aq^i$, where $A = e^c$, $q = e^k$.

Number of cities	c	k	A	q	$r=1/q$
45	13.36066	-0.045002	634542.788	0.955996	1.04602995

In summary, the following conclusions can be made:

1. Development of cities of Central Asia can be well explained by the Weber-Fechner Law (see table A5.3.6 in the Appendix).
2. Weber constant is equal to 0.045.
3. For the change in the population to be noticeable (for infrastructure, administrative decisions) this change should be greater than 4.6% of the population of the city ($r=1/q=1.046$). Therefore, the decisions (administrative, economic, ecological etc.) should be changed if the population of the cities increases by more than 4.6%.

Change in the Weber coefficients

Estimates of the coefficients of regression $\ln N_i = c + k \cdot i$ based on the data on the population of the Central Asian cities for the years 1970-2006 as well as the coefficients c and k are given in tables A5.3.7 in the Appendix and Table 5.3.8.

Table 5.3.8. *Parameters of regression of logarithms of the population N_i for cities of Central Asia agaist its ranks: $\ln N_i = c + k \cdot i$.*

Years	***c***	***k***
1970	13.21387	-0.06884
1971	13.24355	-0.06884
1975	13.30165	-0.06317
1980	13.32473	-0.05399
1985	13.39433	-0.052
1987	13.40747	-0.04943
1990	13.41749	-0.05069
1999	13.36066	-0.045
2006	13.48998	-0.05169

The following Figure 5.3.7 illustrates the regressions $ln\ N_i = c+k{\cdot}i$ estimated in Table 5.3.8.

Figure 5.3.7. *Regressions $ln\ Ni = c+k{\cdot}i$ for Central Asian cities in 1970-2006*

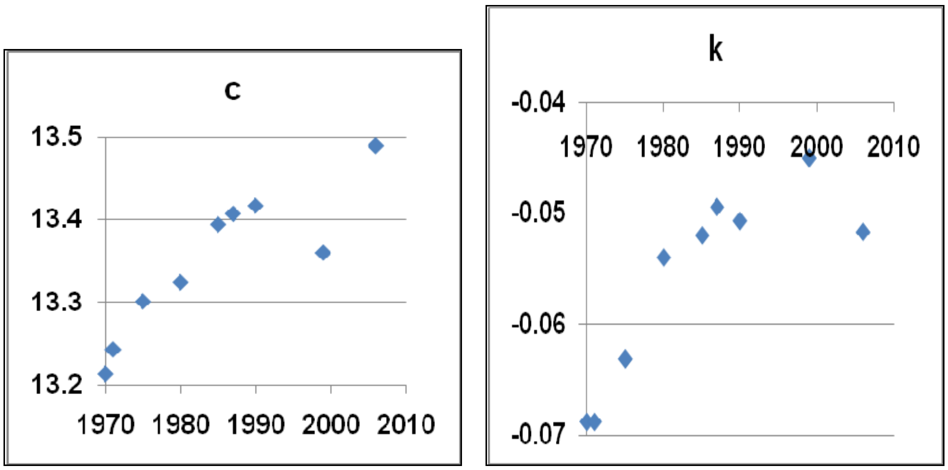


Figure 5.3.8. *Weber relations $N_i = Aq^i$ for Central Asian cities in 1970-2006*

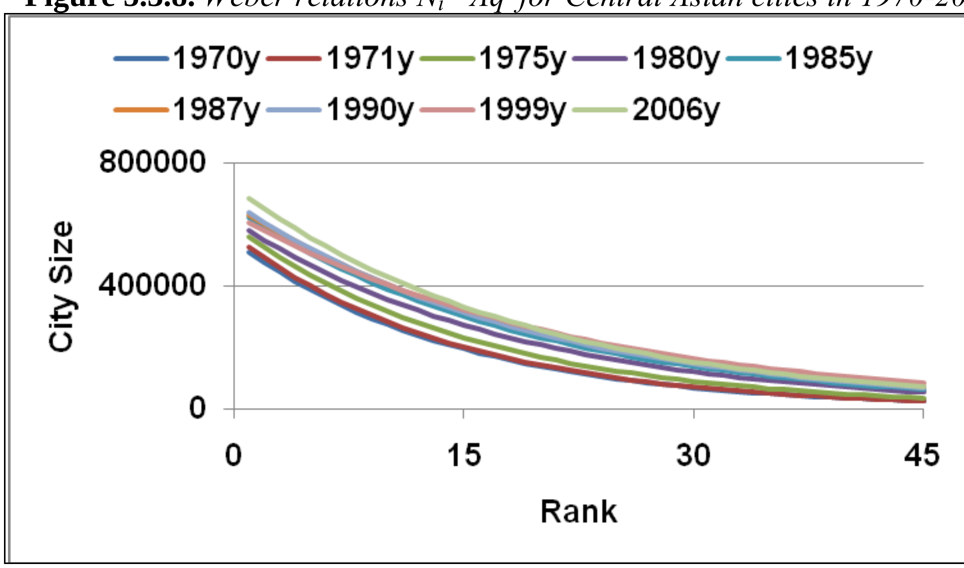


Table A5.3.8 in the Appendix provides the estimation results for the regression of the (estimated) parameters c and k on the time trend (the ranks t of years 1970, 1971, ..., 2006) and the dummy political variable P that takes value 0 prior to the collapse of the USSR in 1991 and value 1 afterwards.

Thus, the estimated regressions are

$c=-7.601766+0.010573\cdot t-0.144598\cdot P$,

$k=-1.877394+0.000919\cdot t-0.011148\cdot P$,

$\ln N_i=-7.601766+0.010573\cdot t-0.144598\cdot P+(-1.877394+0.000919\cdot t-0.011148\cdot P)\cdot i$

,

$N_i=\exp(-7.601766+0.010573\cdot t-0.144598\cdot P)\cdot\exp((-1.877394+0.000919\cdot t-0.011148\cdot P)\cdot i)$

that is $N_i=A(t,P)\cdot q(t,P)^i$, where $A(t,P)=\exp(-7.601766+0.010573\cdot t-0.144598\cdot P)$,

$q(t,P)=\exp(-1.877394+0.000919\cdot t-0.011148\cdot P)$.

Consequently, the disintegration of the USSR led to a decrease in the growth of cities in Central Asia. Apparently this is due to the emigration of non-indigenous people in other countries.

Caucaus

Estimates of the coefficients of regression $\ln N_i=c+k\cdot i$ based on the data on the population of the Caucasus cities for the year 2007 as well as the coefficients of the equation $N_i=Aq^i$ are given in tables A5.3.9 in the Appendix and Table 5.3.9.

Table 5.3.9. *Estimates for the regression* $\ln N_i=c+k\cdot i$ *and the implied relation* $N_i=Aq^i$ *for cities of Caucasus Asia in 2007*

Number of cities	c	k	A	q	$r=1/q$
10	14.50335	-0.336194	1989412.65	0.714484	1.39961052

In summary, the following conclusions can be made:

1. Development of cities of Caucasus can be explained by the Weber-Fechner Law (see Table A5.3.9 in the Appendix).
2. The Weber constant is equal to 0.336.
3. For the change in the population to be noticeable (for infrastructure, administrative decisions) this change should be greater than 39.96≈40% of the population of the city ($r=1/q=1.39961$). Therefore, the decisions (administrative,

economic, ecological etc.) should be changed if the population of the cities increases by more than 40%.

Changes in the Weber coefficients

Estimates of the coefficients of regression $\ln N_i = c + k \cdot i$ based on the data on the population of the Caucasus cities for the years 1970-2007 as well as the coefficients of the equation $N_i = Aq^i$ are given in tables A5.3.10 in the Appendix and Table 5.3.10.

Table 5.3.10. *Estimates of the parameters **c** and **k** in the regression ln $N_i = c + k \cdot i$ for the cities in the Caucasus in* 1970-2007

Years	*c*	*k*
1970	13.86252	-0.276535
1971	13.88072	-0.274877
1975	13.98215	-0.257304
1980	14.12501	-0.2618
1985	14.21326	-0.26222
1987	14.24711	-0.262647
1990	14.29858	-0.299119
2007	14.50335	-0.336194

Figure 5.3.9. *Parameters **c** and **k** of regression ln $N_i = c + k \cdot i$ for cities of Caucasus in 1970-2007*

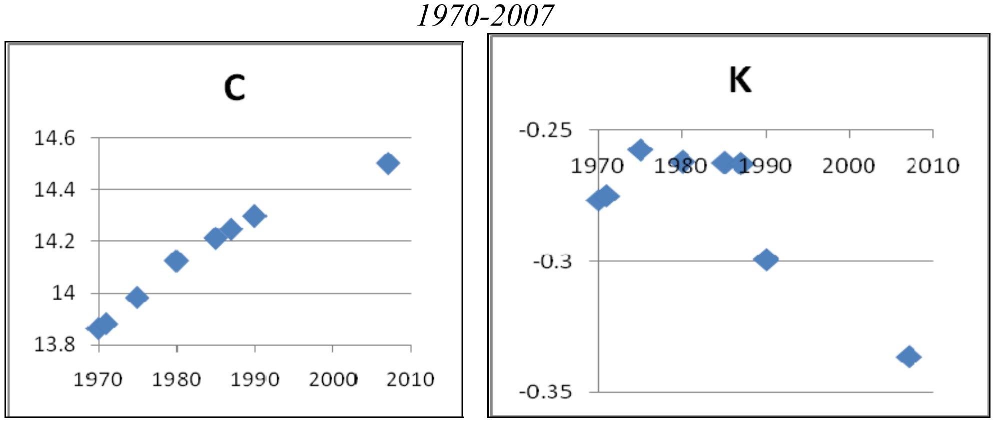
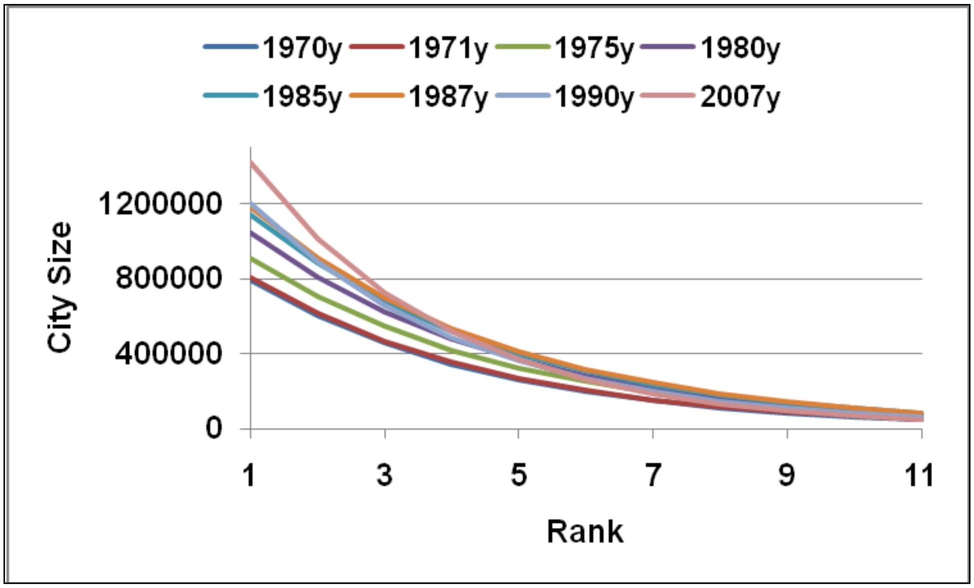


Table A5.3.11 in the Appendix provides the estimation results for the regression of the (estimated) parameters *c* and *k* on the time trend (the ranks *t* of years 1970, 1971,

..., 2006) and the dummy political variable P that takes value 0 prior to the collapse of the USSR in 1991 and value 1 afterwards.

The following Figure 5.3.10 illustrates the regressions $\ln N_i = c+k\cdot i$ estimated in Table A5.3.11 in the Appendix.

Figure 5.3.10. *Weber relations $N_i = Aq^i$ for the cities in the Caucasus in 1970-2007*



Thus, the estimated regressions are

$c = -30.22902 + 0.022385 \cdot t - 0.194493 \cdot P$, $\quad k = -0.270643 - 0.065551 \cdot P$,

$N_i = \exp(-30.22902 + 0.022385 \cdot t - 0.194493 \cdot P) \cdot \exp((-0.270643 - 0.065551 \cdot P) \cdot i)$,

that is $N_i = A(t,P) \cdot q(t,P)^i$, where $A(t,P) = \exp(-30.22902 + 0.022385 \cdot t - 0.194493 \cdot P)$,

$q(t,P) = \exp(-0.270643 - 0.065551 \cdot P)$.

Consequently, the disintegration of the USSR led to a decrease in the growth of cities in the Caucasus. Apparently this is due to the emigration of non-indigenous people in other countries.

5.3.3. Hierarchy of logarithms

Though communication of type of Weber-Fechner between quantity of inhabitants of cities and their ranks is comprehensible from the point of view of the statistical importance, specification of a kind of dependence is desirable. It has appeared possible to be made by means of hierarchy of logarithms in the regress equation.

Ruissia

We will designate: $\ln^4(\cdot)=\ln(\ln(\ln(\ln(\cdot))))$. In Figure 5.3.11 sites of cities of Russia on a scale are resulted "Rank - $\ln^4$(Population)".

Figure 5.3.11. *Russian cities in 1897-2009 (Except for Moscow and Saint-Petersburg)*

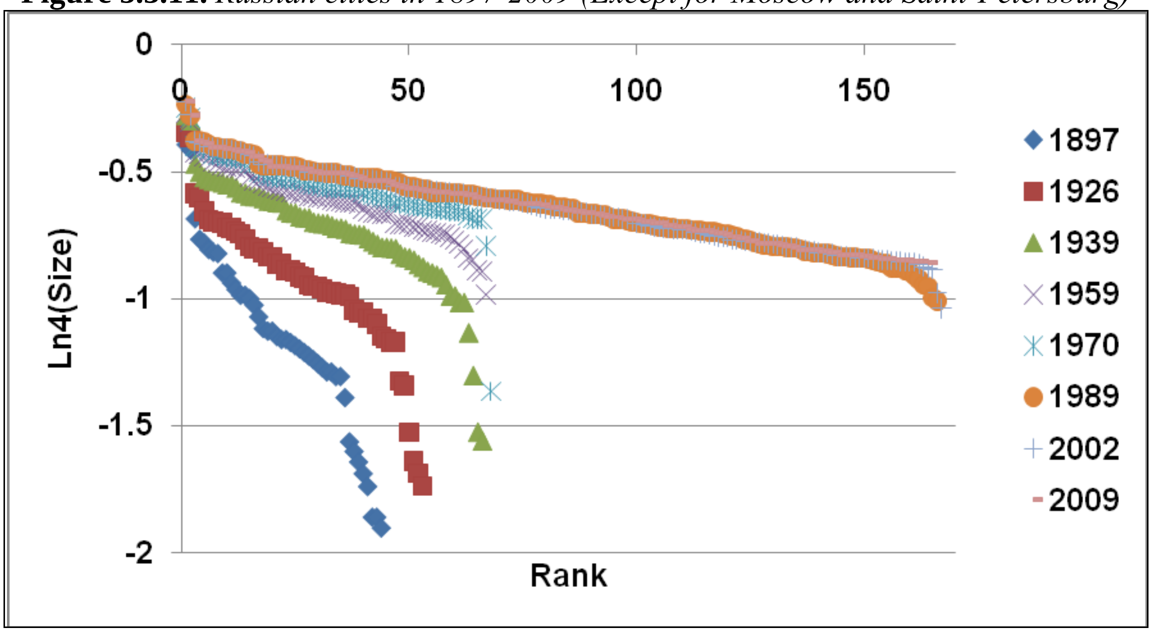
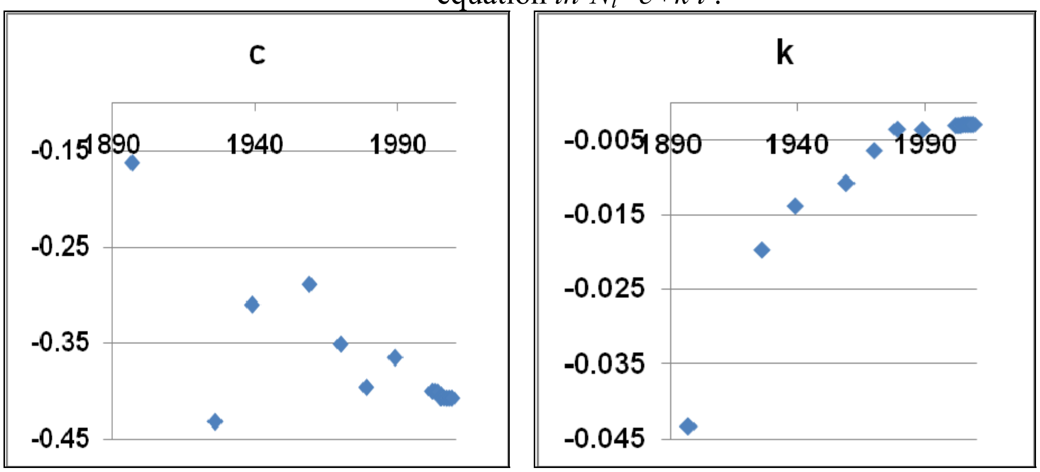


Table 5.3.11. *Estimates of the parameters* ***c*** *and* ***k*** *in the regression* $ln^4 N_i = c_4 + k_4 \cdot i$ *for the cities in Russia in* 1897-2007

Yeares	c_4	k_4
1897	-0.163	-0.04339
1926	-0.43223	-0.01968
1939	-0.31021	-0.01384
1959	-0.28938	-0.01074
1970	-0.35158	-0.00642

1979	-0.39621	-0.00351
1989	-0.3655	-0.00358
2002	-0.40034	-0.003
2003	-0.40061	-0.003
2004	-0.4012	-0.00299
2005	-0.40728	-0.00287
2006	-0.40728	-0.00287
2007	-0.40773	-0.00287
2008	-0.40767	-0.00287
2009	-0.40766	-0.00287

Figure 5.3.12. Change of coefficients c and k in the years 1897-2009 in the regression equation $ln^4N_i=c+k{\cdot}i$.



Estimates of the coefficients of regression $\ln^4 Ni = c+k{\cdot}i$ based on the data on the population of the Russian cities for the years 1897-2009 and are given in tables A5.3.12, A5.3.13 in the Appendix.

According to the information given in tables A5.3.12, A5.3.13, the population of cities N_i and their ranks i are regressed in the equation

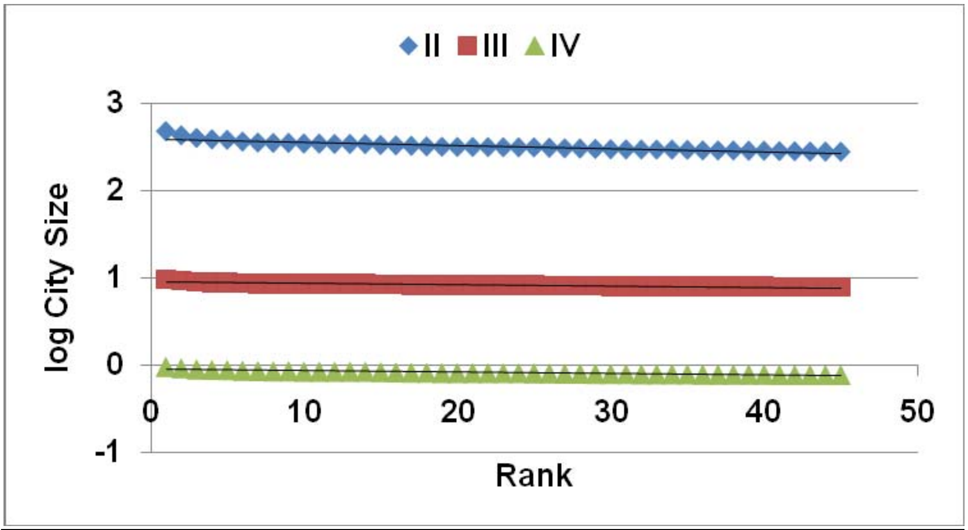
$$\ln^4 N_i = c(t) + k(t)\cdot i = -0.258998 - 0.001264\cdot t + (-0.071110 + 0.014469\cdot \ln t)\cdot i$$

$$N_i = \exp(\exp(\exp(\exp(-0.258998 - 0.001264\cdot t + (-0.071110 + 0.014469\cdot \ln t)\cdot i))))$$

where t=0,1,2,... since 1890.

Central Asia

Figure 5.3.13. *Rank-Population diagrams for different logarithm powers in the hierarchy of logarithms for cities of Central Asia in 1999*



Note. (II) – $\ln^2(N_i)$, (III) - $\ln^3(N_i)$, (IV) - $\ln^4(N_i)$, where $\ln^r(\cdot)$ means the r-th iterations of logarithms.

Estimates of the coefficients of regression $\ln^r Ni = c+k\cdot i$ based on the data on the population of the Central Asian cities in 1999 and are given in table A5.3.14 in the Appendix.

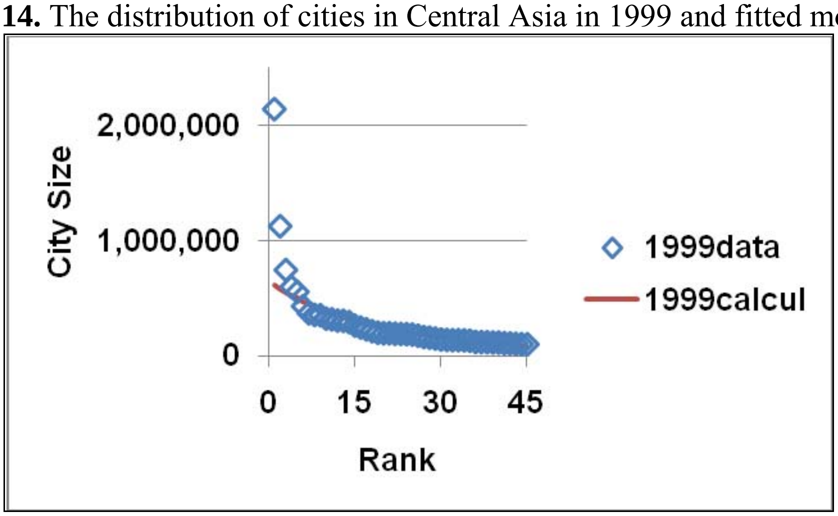
According to Table A5.3.14 the best in all respects is the model

$$\ln^4(N_i) = -0.048076 - 0.001534 \cdot i\,,$$

$$N_i = \exp(\exp(\exp(\exp(-0.048076 - 0.001534 \cdot i)))) \,. \qquad (5.3.1)$$

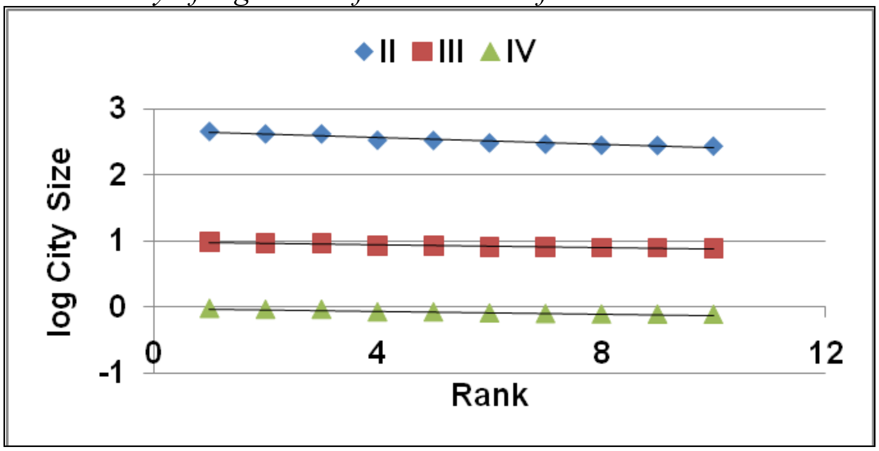
This model describes well the distribution of all cities in Central Asia except the three outliers of Tashkent, Almaty and Bishkek (see Figure 5.3.14).

Figure 5.3.14. The distribution of cities in Central Asia in 1999 and fitted model (5.3.1)



Caucasus

Figure 5.3.15. *Rank-Population diagrams for different logarithm powers in the hierarchy of logarithms for the cities of the Caucasus in 2007*



Note. (II) – $\ln^2(N_i)$, (III) - $\ln^3(N_i)$, (IV) - $\ln^4(N_i)$, где $\ln^r(\cdot)$ means r iterations of logarithms.

Estimates of the coefficients of regression $\ln^r Ni = c+k\cdot i$ based on the data on the population of the Caucasus cities in 2007 and are given in table A5.3.15 in the Appendix.

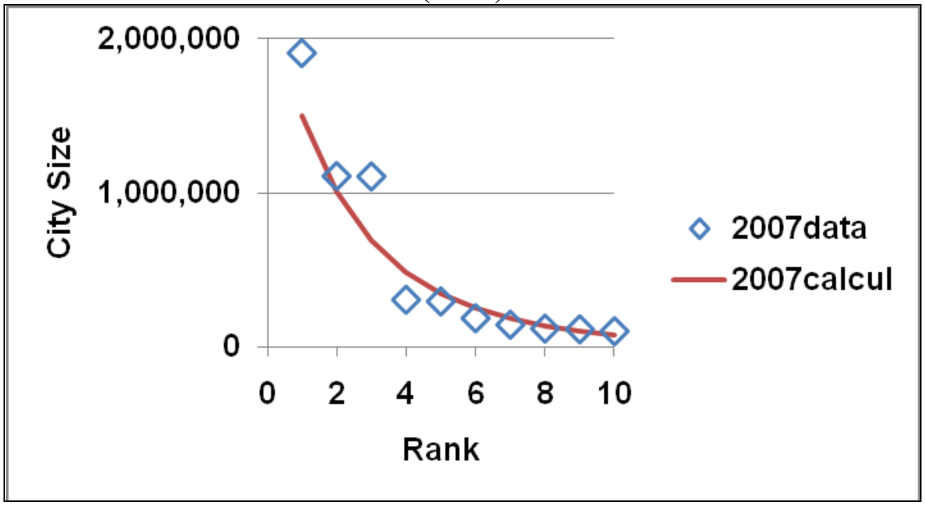
According to Table A5.3.15 the best in all respects is the model

$$\ln^4(N_i) = -0.013023 - 0.010991 \cdot i\,,$$

$$N_i = \exp(\exp(\exp(\exp(-0.013023 - 0.010991 \cdot i)))) \,. \qquad (5.3.2)$$

This model describes well the distribution of all cities in Central Asia except the outlier of Baku (see Figure 5.3.16).

Figure 5.3.16. The distribution of cities in the Caucasus by rank in 2007 and fitted model (5.3.2)



Therefore we can conclude that:

1. The distribution of the size of the largest cities of Russia, Belarus, Central Asia and Caucasus is consistent with Zipf's law.
2. The distribution of the size of the size (all) cities of Russia, Belarus, Central Asia and Caucasus satisfies the law of Weber-Fechner except the largest Megapolyus.
3. The Great October Revolution and World War II led to an increase in Russian cities due to influx of rural population in the city. When Stalin began forced urbanization, people from villages in the 30 th, 40 th, 50 th years, went into the city.
4. The collapse of the USSR led to a relative reduction cities of Central Asia and Caucasus as a result of relocation of non-indigenous population in rural areas of Russia. The collapse of the USSR at the rate of urban growth in the Belarus statistically significant effects are not influence.
5. Distribution of cities in Russia, Belarus, Central Asia and Caucasus is best described by models based on the hierarchy of the logarithms of their sizes.

5.4 Results concerning the "within distribution" city dynamics

5.4.1 Markov chains analysis

In this section, we apply Markov chains analysis to study a movement speed and form of convergence within the city size distribution. We employ data on population of all cities for Belarus, Hungary, Poland, and for 479 of Russia (out of 1037 cities according to 2002 census). The dataset is described in Table A5.4.1 in the Appendix.

The main sources of the detailed city data are the national official statistical information services of CEE and CIS countries. Data in national statistics are presented for census years as well as estimates on the beginning of the corresponding year. The number of cities and other characteristics of urban systems of Belarus, Hungary, Poland, and Russia are described in the Table 5.4.1.

Table 5.4.1. The main description of the data by countries.

Indicator	Belarus			Poland		
	1970	1989	2009	1970	1989	2009
Number of cities	198	202	206	802	828	890
Urban pop. (ths)	3886.9	6768.5	7148.5	18492.7	23455.3	23279.4
Size of a min city	1.2	0.7	0.6	1	1.2	0.9
Average city size	19.6	33.5	34.7	23	28.3	26.2
Size of a max city	907.1	1612.8	1829.1	1387.8	1651.2	1709.8

Table 5.4.2. (continuation)

Indicator	Hungary			Russia		
	1970	1989	2001	1970	1989	2007
Number of cities	237	237	237	479	479	479
Urban pop. (ths)	6124.3	6741.1	6415.7	52971.1	69437.2	77927.7
Size of a min city	0.68	1.1	1.4	1.9	1.3	1.15
Average city size	25.8	28.4	27	110.6	145	197
Size of a max city	1945.1	1934.8	1712.7	7063	8769.1	10126.4

In order to carry out the methodology described in section 4.4, we should choose a discretization of the cities' sizes. As pointed out by Magrini (1999), an improper discretization may have the effect of removing the Markov property and therefore may lead to misleading results, especially as is in our case when computations of ergodic distributions are based on the estimates of the discrete transition probabilities. Quah (1993) and Le Gallo (2004) choose to discretize the distribution in such a way that the initial classes include a similar number of elements. Cheshire and Magrini (2000) base their choice between possible classes in terms of the ability of the discrete distribution to approximate the observed continuous distribution.

In our study following the paper of Le Gallo and Chasco (2009), we have tried different ways of discretizing the distribution, divided it on 5, 6 and 7 classes. We chose Poland to check possible distributions providing we have the biggest dataset for this country (890 cities) and this country is one of the most successful among transition economies. Final discretization should be chosen by considering the best performance of the test for order one for all countries' city distributions.

The assumption of a first-order stationary Markov process requires the transition probabilities, p_{ij}, to be of order 1, that is, to be independent of classes at the beginning of previous periods (at time t − 2, t − 3, …). If the chain is of a higher order, the first-order transition matrix will be misspecified. Indeed, it will contain only part of the information necessary to describe the true evolution of population distribution. Moreover, the Markov property implicitly assumes that the transition probabilities, p_{ij}, depend on i (i.e., that the process is not of order 0).

In order to test this property, Bickenbach and Bode (2003) emphasize the role of the test of time independence. In determining the order of a Markov chain, Tan and Yilmaz (2002) suggest, firstly, to test order 0 versus order 1; secondly, to test order 1 versus order 2; and so on. If the test of order 0 against order 1 is rejected, and the test of order 1 against order 2 is not rejected, the process may be assumed to be of order 1.

After trying different variants we decided to divide all cities on seven classes: 1) population less than 10% of the countries' average, 2) population between 10 and 20% of the average 3) population between 20 and 30% of the average, 4) population between 30 and 50% of the average, 5) population between 50 and 100% of the average, 6) population between 100 and 200% of the average, and 7) population more than 200% of the average. This division appears to give relatively balanced distribution for all four countries.

However the way of cities' division on classes could be changed after considering the performance of the test for Markovity of order one for all countries with detailed data. We can get different results of that test for different countries and this will give us information about a possibility to build more balanced classes at some cost to this test for certain countries.

To test for order 0, the null hypothesis $H_0: \ \forall\, i = 1,\ldots,K \ p_{ij} = p_i$ is tested against the following alternative $H_a : \exists\, i \in \{1,...,K\} \quad p_{ij} \neq p_j$. The appropriate likelihood ratio (LR) test statistic reads as follows:

$$LR^{(O(0))} = 2\sum_{i=1}^{K}\sum_{j\in A_i} n_{ij}(t)\ln\frac{\widehat{p_{ij}}}{\widehat{p_i}} \sim \chi^2\left((K-1)^2\right),$$

assuming that $\widehat{p_i} > 0$, $\forall\, i \in \{1,..,K\}$, $A_i = \{j : \widehat{p_{ij}} > 0\}$ is the set of nonzero transition probabilities under *Ha*

To test for order 1 versus 2, a second-order Markov chain is defined by also taking into consideration the population size classes in which the cities were at time $t-2$ and assuming that the pair of successive classes *k* and *i* forms a composite class. Then, the probability of a city moving to class *j* at time *t*, given it was in *k* at $t-2$ and in *i* at $t-1$, is p_{kij}. The corresponding absolute number of transitions is n_{kij}, with the marginal frequency being $n_{ki}\left(t-1\right) = \sum_j n_{kij}(t-1)$ To test $H_0 : \forall\, k \in \{1,...,K\} \quad p_{kij} = p_{ij}$ against $H_a : \exists k \in \{1,...,K\} \quad p_{kij} \neq p_{ij}$, the $\widehat{p_{kij}} = \frac{n_{kij}}{n_{ki}}$, where $n_{kij} = \sum_{t=2}^{T} n_{kij}(t)$ $n_{ki} = \sum_{t=2}^{T} n_{ki}(t-1)$. The p_{ij} are estimated from entire data set as $\widehat{p_{ij}} = \frac{n_{ij}}{n_i}$. Appropriate LR test statistic reads as follows:

$$LR^{(O(1))} = 2\sum_{k=1}^{K}\sum_{i=1}^{K}\sum_{j\in C_{hi}} n_{kij}(t)\ln\frac{\widehat{p_{kij}}}{\widehat{p_{ij}}} \sim \chi^2\left(\sum_{i=1}^{K}(c_i-1)(d_i-1)\right).$$

Similar to the notation above, $C_i = \{j : \widehat{p_{ij}} > 0\}$, $c_i = \#C_i$, $C_{ki} = \{j : \widehat{p_{kij}} > 0\}$, and $d_i = \#\{k : n_{ki} > 0\}$. In our case $K = 7$.

If both Markovity of order 0 and of order 1 are rejected, the tests can be extended to higher orders by introducing additional dimensions for population size at time $t-3$, $t-4$, and so on. However, since the number of parameters to be estimated increases exponentially with the number of time lags, while the number of available observations decreases linearly for a given data set, the reliability of estimates and the power of the test decrease rapidly. Therefore, Tan and Yilmaz (2002) suggest setting an a priori limit up to which the order of the Markov chain can be tested.

All results of testing Markovity for every country one can observe in Appendix Table A.5.4.2 – Table A.5.4.10. In our case most data passed the tests for Markovity of order greater or equal to one.

For instance, see Table A.5.4.2. (Poland), Markovity of order 0 is tested using test statistic (5.4.1) at every moment t= 1961, 1974, 1985, 1994, 2004 (in our investigations parameter t runs by decades, or approximately by decades depending on lack of data on some country). The result $LR^{(O(0))}(1961) = 1943.578$, prob=0, df=36 leaves no doubt that the process strongly depends on the initial condition at time t-1. That is the chain is at least of order 1. Applying the test statistic (5.4.2.2) to the same moments of time we get the result: $LR^{(O(1))}(1964) = -396.545$, prob=0, df=28 indicating about Markovity of order 1 and higher. As we mentioned above we cannot continue test of Marcovity order 1 versus 2 etc., because of exponential growth of parameters to be estimated with having bounded data.

Received Markovity test results for all countries with detailed data mean that we do not need to perform a revision of the discretization of cities on classes for Markov chains estimation procedure.

Tables A.5.4.11. – A.5.4.14. contain the first-order transition probability matrices with the ML estimates p_{ij} of the transition probabilities for population in Poland, Belarus, Hungary, and Russia.

Note that all transition probability matrices for studying countries are regular. Matrices let us draw conclusions on intensity of interclass movements. Using those matrices according to methodology described, we can extract information related to cities' mobility speed and convergence pattern.

For example, in Poland during the half of a century, there were 459 instances of a city having a population size lower than 10 percent of the average. The majority of these cities (78.6%) remained in that size class at the end of the decade, while 15.5% moved up one class by the end of the decade.

The high probabilities on the diagonal in all countries show a low interclass mobility, i.e., a high-persistence of cities to stay in their own class from one observation to another over the whole period. Eaton and Eckstein (1997) interpret diagonal elements of the transition approaching 1 as parallel growth. Since these elements are not exactly 1, we can analyze the propensity of cities in each cell to move into other cells. In particular, it appears that the largest and smallest cities (classes 1 and 7, respectively) have higher persistence while medium-sized cities (categories 3, 4 and 5) have more probability of moving to smaller categories. In classes 2 and 3 a small number of cities if any move up to higher categories more than two steps. Only in case of Poland in classes 2 and 3 the probability of moving up a class exceeds that of moving down. In Belarus the probability of moving down a class exceeds that one in other countries.

This low inter-class mobility of cities is in line with the results found for other cases such as US MSA's (Black and Henderson 2003) and all Spanish municipalities (Le Gallo and Chasco 2009).

Then, in order to determine the speed with which the cities move within the distribution, we consider the matrix of mean first passage time M_P, where every element indicates the expected time for a city to move from class i to class j for the first time (Tables A.5.4.15 – A.5.4.2.18). M_P is defined as (Kemeny and Snell 1976, Chap. 4):

$$M_P = (I - Z + \mathbf{1}\, Z_{dg})D$$

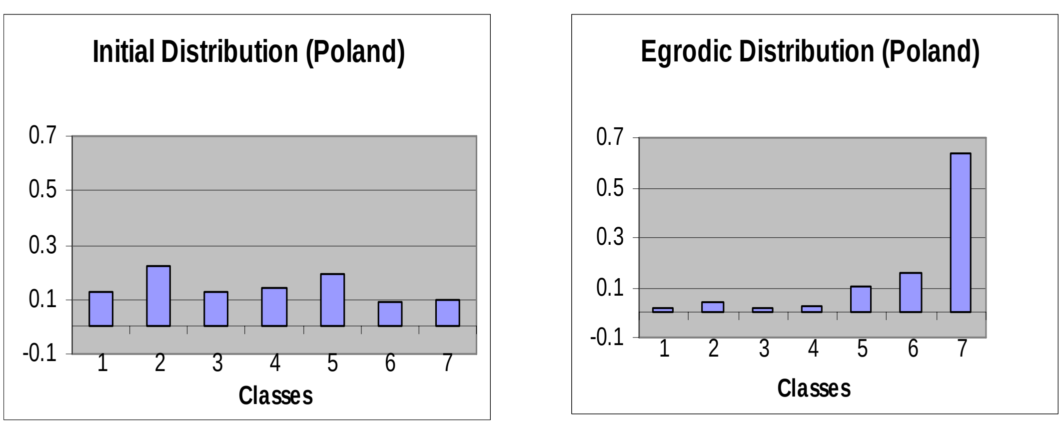
where I is the identity matrix, $Z = (I - M + M^*)^{-1}$, M is the probability transition matrix, $M^* = \lim_{n\to\infty} M^n$, $\mathbf{1}$ is a matrix of ones, Z_{dg} results from Z setting off-diagonal entries to 0, and $D = Diag\left\{\frac{1}{m_1^*},...,\frac{1}{m_K^*}\right\}$, $m_1^*,..,m_K^*$ are elements of M^*.

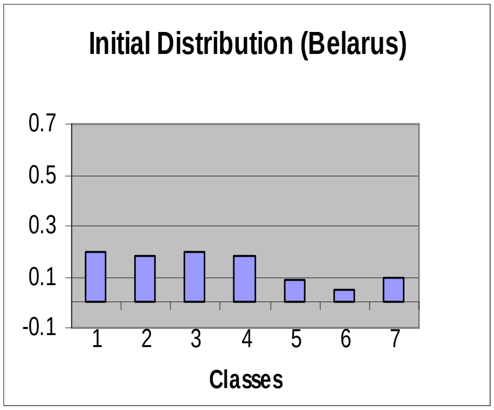
For example, the expected time for Belarusian city to move from class 1 to class 2 is equal to 220 years, while the moving from 2 to 1 will happened in 99 years. In whole the mean number of years to reach any class is relatively high: for example, the shortest time passage for Poland is 115 years (move from class 1 to class 3) and the longest is 6060 years (move from class 7 to class 1). We should remember that these calculations account for the fact that starting from class 4, a city might visit classes 6, 5, 3, 2 or 1 before going to class 7.

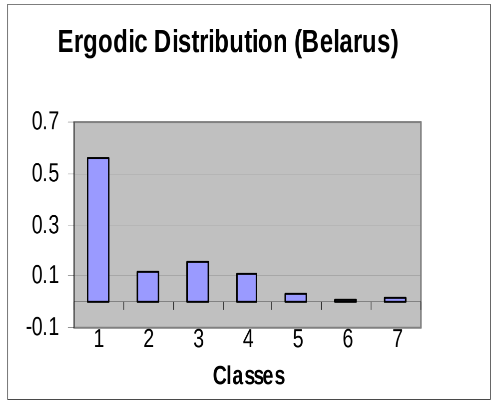
Belarusian matrix shows the passage from higher class to lower one is more probable than from lower to higher. That is not the truth for Polish and Hungarian cities where the moving to higher class is faster. For example, for Belarusian cities to first visit class 7 from class 1 it takes 40077 years, while for Polish and Hungarian it takes 827 and 8168.8 years respectively. On the contrary, to first visit class 1 from class 7 it takes 1190 years for cities in Belarus, while for Poland and Hungary it takes 6060 and 2195 years respectively. In Belarusian and Russian matrices all upper diagonal elements greater than lower diagonal ones. That means Belarusian and Russian cities tends from higher class to lower one. All upper diagonal elements of Polish matrix less that lower diagonal ones. That is all Polish cities tend to move from lower classes to higher ones. The situation with Hungarian matrix is a bit different. We can see that more probable moves from lower classes to higher ones take place up to third class (upper diagonal elements less than lower ones). From fourth to seven classes we can see backward moves (upper diagonal elements greater than lower ones). Comparing with results of Le Gallo and Chasco (2009), obtained for Spanish urban system we may say that maximal entry of the mean first passage matrix is 3110,7 years. It corresponds to a mean first time passage of a city from first class to last (sixth) class. Moves happen more probably between neighbor classes. Minimal time to move between classes is 91.9 years. It is a transition from class 5 to class 4.

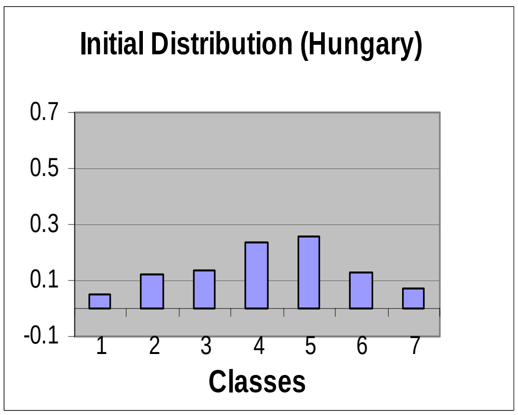
The difference in the models of urban system development and the forms of cities' convergence for Belarus, Russia on the one part and Poland and Hungary on the other part becomes obvious after comparison of initial versus ergodic distribution pattern matching (Tables A.5.4.19. – A.5.4.22) or see Figure 5.4.1.

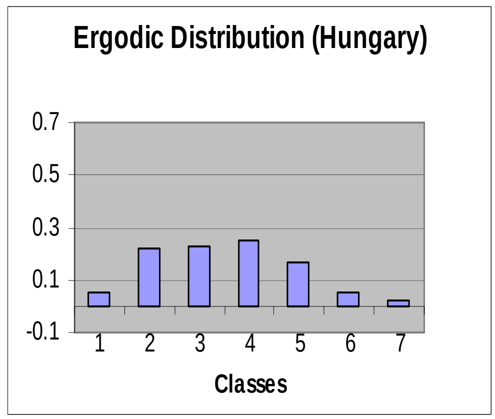
Figure 5.4. 1 Initial and ergodic distribution of cities' sizes in Poland, Belarus, Hungary, and Russia

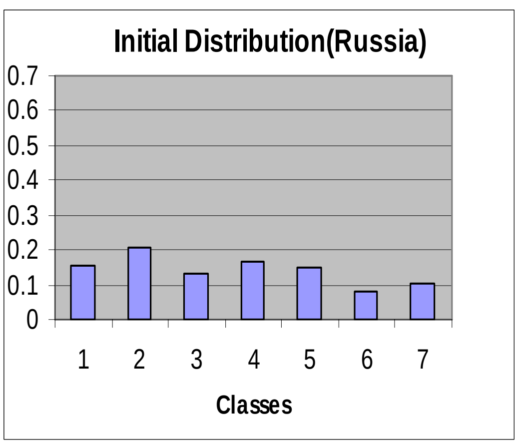


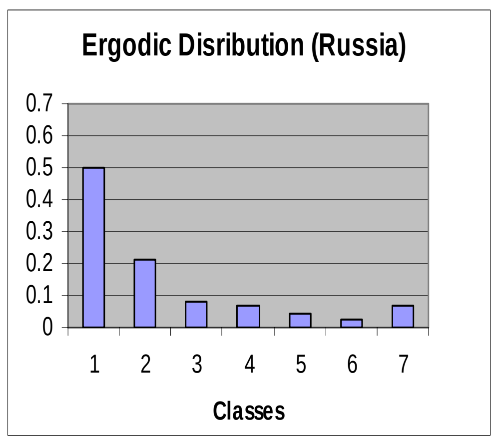








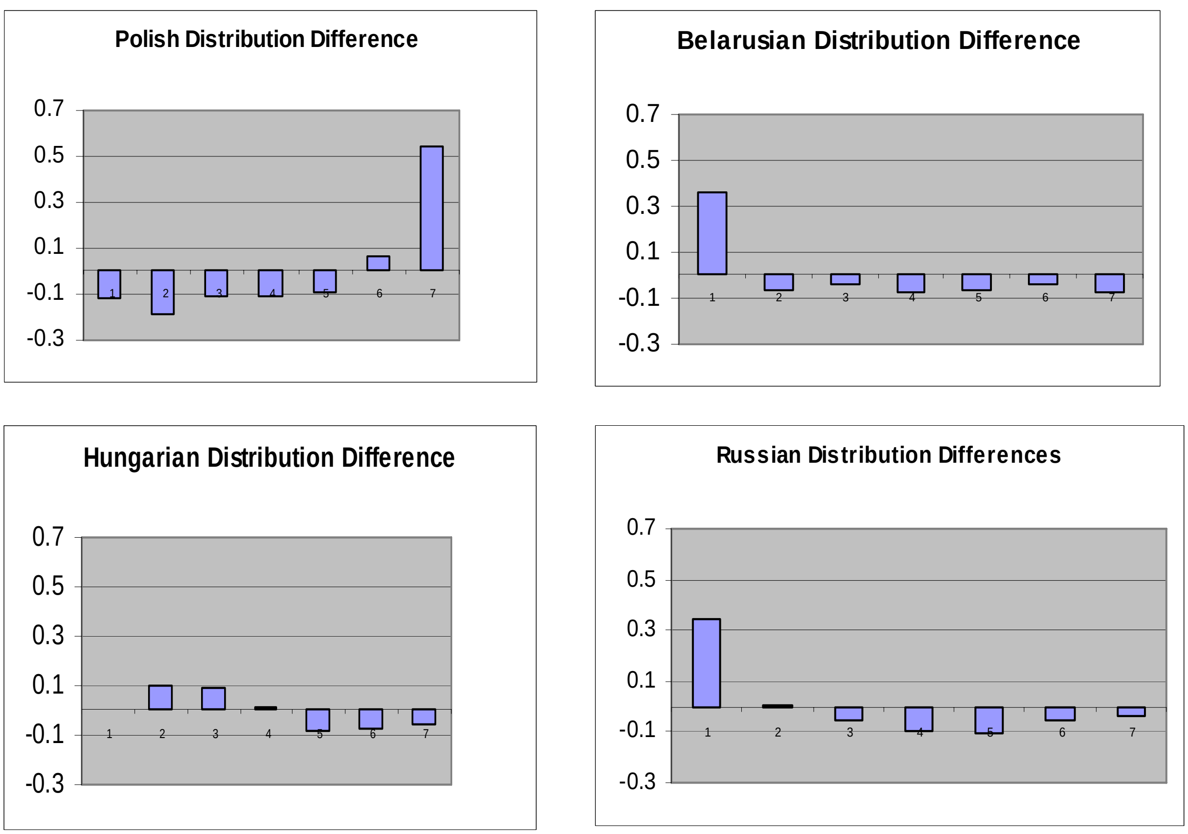




The ergodic distribution can be interpreted as the long-run equilibrium city-size distribution in the urban system. Given a regular transition matrix, with the passage of many periods, there will be a time where the distribution of urban system will not change any more: that is the ergodic or limit distribution. It is used to assess the form of convergence in a distribution. Concentration of the frequencies in a certain class would imply convergence (if it is the middle class, it would be convergence to the mean), while concentration of the frequencies in some of the classes, that is, a multimodal limit distribution, may be interpreted as a tendency towards stratification into different convergence clubs. Finally, a dispersion of this distribution amongst all classes is interpreted as divergence.

The results for Poland, Belarus, Hungary and Russia are reported on the histograms of Figures 5.4.1., 5.4.2., A.5.4.1. and demonstrate significant differences among countries. For Belarus and Russia it appears that the ergodic distribution is more concentrated in the small and lower middle-size cities (1st to 4th classes), a result that reveals the existence of convergence towards smaller size populations. For Poland it appears that the ergodic distribution is more concentrated in the middle and big-size cities (5th to 7th classes). At the same time, one can see that a level of stability of ergodic distribution compared to the initial one for Belarus, Poland, and Russia is low, while it is relatively more stable for Hungarian distribution. The Figure 5.4.2. shows quantitative difference between ergodic and initial distributions.

Figure 5.4.2. Difference between Initial and Ergodic distributions of cities' sizes in Poland, Belarus, Hungary, and Russia



As one can see Belarus and Russia evolves to the country of small cities, while Poland and Hungary to the country of big and medium sized cities respectively. Studying probability transition matrices and mean first passage time matrices of investigated countries we may say something about movements of cities within the distribution. In case of Hungary probability (see Table A.5.4.13.) to pass from 1 class to 2 four times bigger than from 2 to1, probability to pass from 3 class to 2 is greater than that from 2 to 3. That is cities from 1 and 3 class will move to second one. Furthermore, cities from 7 class will probable to move to 6, cities from 6 class more probable to move in 5 and so on.

Our results for initial and ergodic distributions are comparable with those for Spanish municipalities obtained by Le Gallo and Chasco (2009). Their study shows slightly downward convergence to the second and third classes and is similar to Hungarian pattern.

It may be interesting to represent the differences in the forms of distributions in numerical quantities. We may compare ergodic, initial distributions, and their difference with help of kurtosis statistics: $Kurt(X)=\frac{n\sum_{i=1}^{n}\left(x_i-\overline{X}\right)^4}{\left(\sum_{i=1}^{n}\left(x_i-\overline{X}\right)^2\right)^2}-3$,

that is close to zero if X close to symmetric Gaussian distribution, and far from zero otherwise. The bigger kurtosis the more sharp the peak of X distribution. In terms of shape, such distribution has a more acute peak around the mean (that is, a lower probability than a normally distributed variable of values near the mean) and fatter tails (that is, a higher probability than a normally distributed variable of extreme values). A distribution with negative excess kurtosis is more "broad". In terms of shape, a such type of distribution has a lower, wider peak around the mean (that is a curve of such distribution is mostly convex upward) and thinner tails (that is a curve of such distribution has a narrow domain where it is convex downward). Table A.5.4.24 depicts values of the kurtosis across all countries and shows that Hungarian ergodic and initial distributions are most balanced among all countries. Therefore, we propose to consider Hungarian urban system distribution as a benchmark for assessment of deviations of

Belarusian, Russian and Polish ones. It is clear from the table that all countries initially had low kurtosises. However, magnitudes of kurtosis for ergodic distributions changes and we may arrange countries in order by growing urban pattern starting from country with worse urban ergodic distribution: Belarus =6.8 (with mean value at first class), Russia=5.8 (with mean value at first class), Poland =5 (with mean value at seventh class), and Hungary (with mean value around fourth class). Here the mean value of distributions is significant too.

The influence of space on urban population dynamism by comparing the probability of a city moving down or up in the hierarchy depending whether city is surrounded by towns that contain, on average, less or more population is considered in next subsection.

5.4.2 Studying Spatial Autocorrelation in Belarusian Urban System

To test whether the probability of an upward or downward move of cities is different depending on the urban area context in Belarus we use the following methodology. Let d_{ij} be a distance between city *i* and city *j*. For 207 Belarusian cities they form (207, 207) dimension matrix of distances. We form a spatial weight matrix $w_{ij} = \begin{cases} d_{ij}^{-\alpha}, \text{if} \quad d_{ij} \le c, \\ 0, \quad \text{if} \quad i = j \text{ or } d_{ij} > c \end{cases}$, where c is approximately 150 km (a first quartile of the whole range of distances). The positive parameter α we chose in order to obtain more statistically significant results for spatial autocorrelation. In Le Gallo (2004) and in Cliff and Ord (1981) $\alpha = 2$ because of analogy with Newton's gravitational law. In first considerations we accepted $\alpha = 2$. Then we consider vector-column of dimension (207, 1) with elements $x_i = \begin{cases} 1, \quad \text{if } i \text{ is a growing city}, \\ 0, \quad \text{otherwise} \end{cases}$. Moreover, we considered vector of elements $z_i = x_i - \overline{X}$, where $\overline{X}$ is a sample mean value of X . In Belarus we have 26 such cities in period between 1970 and 2009. To evaluate spatial autocorrelation of upward downward transitions we used Moran's I statistic (Moran, 1950):

$$I = \frac{n}{S_0} \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} z_i z_j}{\sum_{i=1}^{n} z_i^2},$$

where $S_0 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}$, $n = 207$. The empirical value of this statistic is equal $I = -0.0789$, but theoretical expectation value of I under hypothesis of no spatial autocorrelation is equal to $E(I) = -\frac{1}{n-1} = -0.00485$. A standard deviation of Moran's I is equal to 0.254. Consequently, Z-score $\frac{I - E(I)}{sd(I)}$ lays between -1.96 and 1.96 and we cannot reject the null hypothesis of no spatial autocorrelation. Recall that Z-score has Gaussian distribution under the null hypothesis. The consideration of Moran's I statistic for 68 Belarusian diminishing cities gives us an estimation $I = -0.2623$ that is clearly shows negative autocorrelation, but due to big standard deviation of Moran's I we again cannot admit this result at significance level of 5%. However, when we choose $a = 1$ in definition of weighted matrix we get ten times lower standard deviation of the Moran's I statistic sd(I)=0.0206. But the Moran's I is equal to -0.009 for growing cities and -0.015 for vanishing cities. These two estimates of the Moran's I are close to zero and we may say that there is no **global** spatial autocorrelation for all Belarusian cities. Then we apply Geary's C statistic that is more sensitive to **local** spatial autocorrelation (Geary, 1954):

$$C = \frac{n-1}{2S_0} \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} (x_i - x_j)^2}{\sum_{i=1}^{n} (x_i - \overline{X})^2}.$$

The Geary's C varies between 0 and 2. If 0<C<1 than it indicates positive autocorrelation, if 1<C<2 than it means negative autocorrelation, if C=1, than it means no spatial autocorrelation. For growing Belarusian cities C=1.115, sd(C)=0.0309, Z-score=3.73, that means negative local autocorrelation with 0.2% significance level. It indicates that neighboring to growing cities are more dissimilar (diminishing or stable) than expected by chance. That is all growing Belarusian cities geographically tend to be surrounded by neighbors with very dissimilar values. For diminishing cities Geary's C=0.998 and it indicates no spatial autocorrelation.

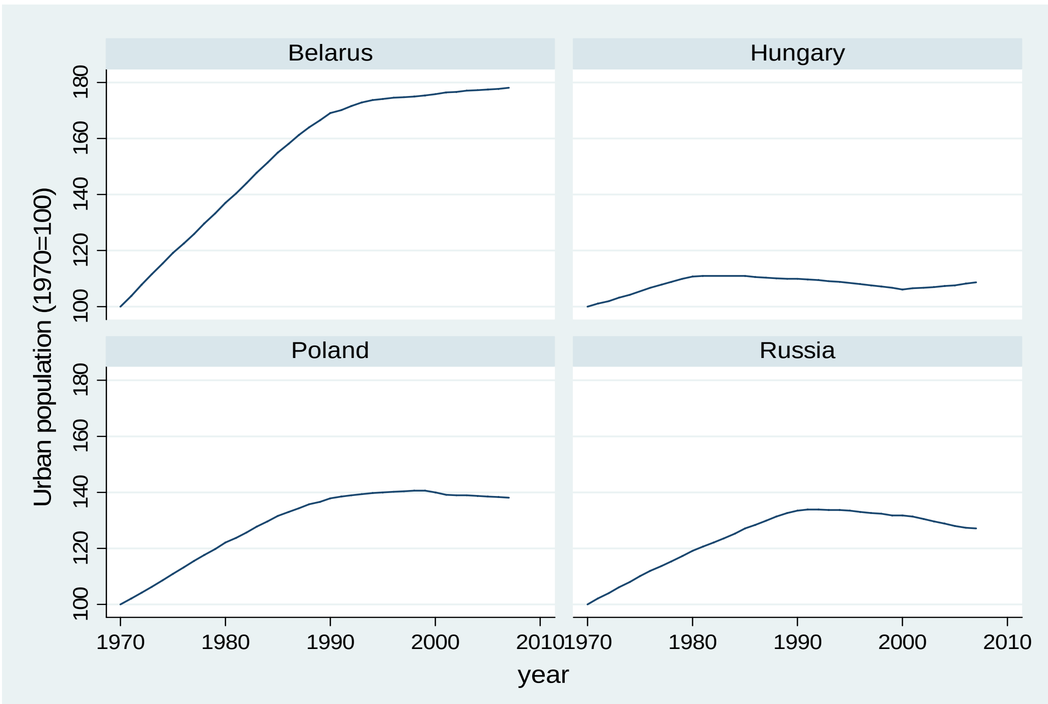
Spatial analysis of Belarusian cities underlines existence of divergence of the urban system in space, not only in time. Negative autocorrelation points to spatial proximity of contrasting values (Anselin and Bera, 1998). That means that there is a tendency for growing Belarusian cities to be surrounded by diminishing cities. It becomes clear if we paint Belarusian map in two colors: red for growing cities and blue for decreasing, see Figure A.5.4.2. On the map we shall see on south a 7th class city Homel surrounded by getting smaller Rechitsa, Kastsukauka, Buda-Kashaliova, Vietka, Uvaravichy, Tserakhauka. On west growing Lida and Byarozauka are surrounded by vanishing Schuchyn, Zhaludok, Radun', Yuratsishki, Dziatlava. The same situation near Brest, Magiliou, Vitsebsk, Polatsk. Only exception is the capital Minsk surrounded by growing Zaslauje, Fanipal', Machulishchy, Lagojsk. Negative autocorrelation indicates that such distribution of Belarusian cities is not by chance. A direction of movement in the population distribution of cities is not independent from the geographic environment. It could be a consequence of a semi-planned economy, where significant state resources are concentrated in the capital (the biggest city) with the rest passed to region centers (another 5 biggest cities) with only small portion allocated to the district level. As a result we have a designed hierarchy of cities or at least the hierarchy which is shaped for the most part not by market forces but rather by visible hand of the state.

This conclusion is supported by the results of Gibrat`s law accepting which demonstrate no strong support of this model of urban system development in the case of Belarus. The presence of doubts in cities proportionate growth in Belarus coincides with our above mentioned results and indicates that the nature of urban systems dynamics in this country is quite specific. Thus to understand this specifics better it is reasonable to make some additional comparisons of the pre and post 1989 development of the examined countries with detailed data. This is a good moment to do this before we will go further trying to investigate the factors that drive the variation of the city size distribution over time.

Studying cities' population, their growth rate dramatic reduction after 1989 becomes obvious. However, this was not the case Belarus in 1989-2007 or in Poland in 1989-1999 where urban population has increased during the mentioned periods (see

Figure A.5.4.3). Changes in population dynamics should obviously have influenced the city size distribution.

Figure 5.4.3. Urban population growth in four transition countries (1970=100)

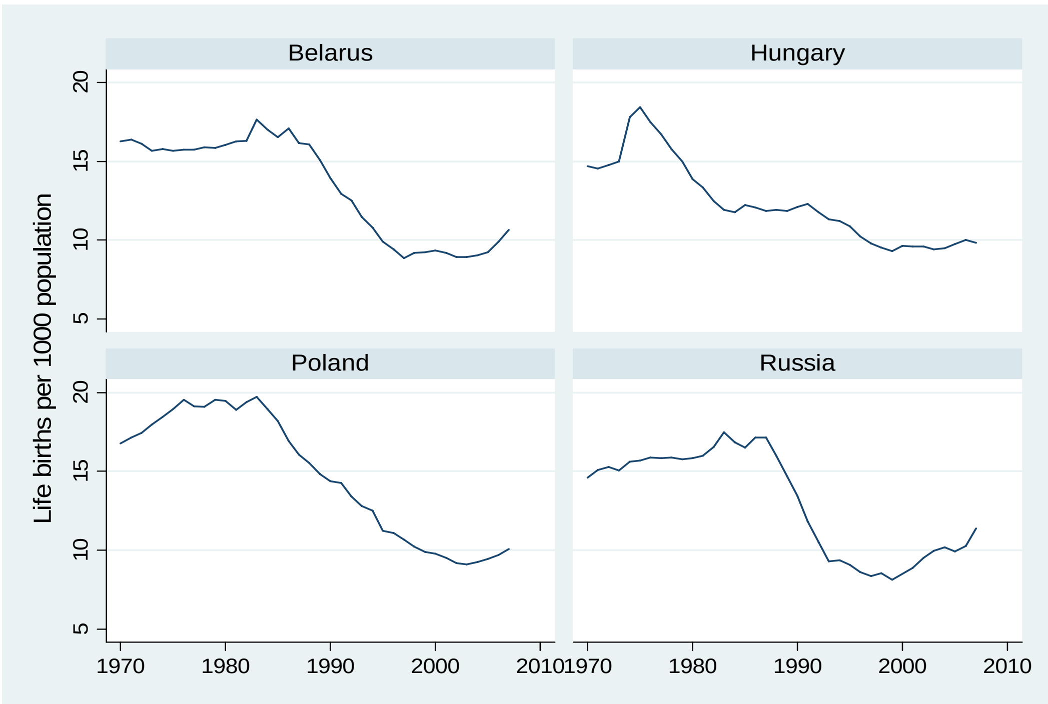


In most transition countries the economic and political reforms at least in the first six years have been accompanied by a rapid impoverishment of large sections of society and increasing uncertainty about the future. According to UNICEF (1994) between 1989 and 1994, marriage rates in transition countries fell by between one-quarter and one-half; birth rates shrank by up to 40 percent and death rates among male adults due to cardiovascular and violent causes often more than doubled. By 1994 the natural increase of the population had become negative in Bulgaria, the Czech Republic, Hungary, Romania, the three Baltic countries, Russia, Ukraine and Belarus.

Below there is an illustration of life births per 1000 population drop in Belarus, Hungary, Poland and Russia (Figure 5.4.4).

One can notice that demographic changes started in the mid 1980s or even 70s in the case of Hungary. It should be noted that, in spite of a similar pattern of life births decline in the first decade after 1989 for the countries in the sample (excluding non-European CIS countries), only Poland demonstrates positive rate of natural population increase (excluding changes due to migration) and negative net external migration at the same time.

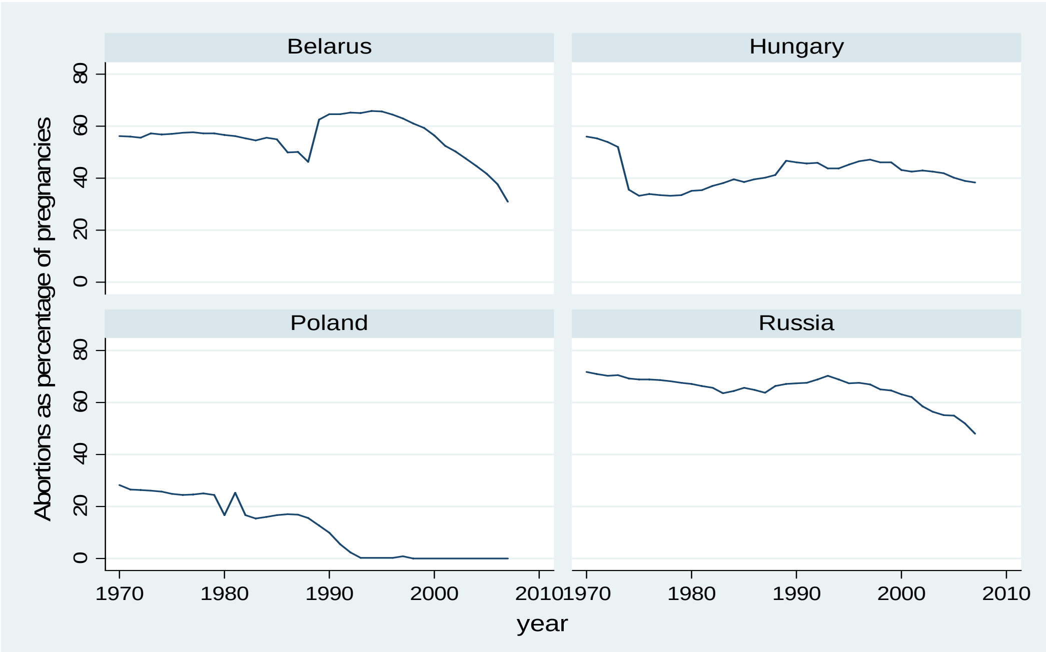
Figure 5.4.4. Life births decline per 1000 population in four transition countries



This may indicate that as opposed to other countries, Polish formal and informal institutions were able to soften economic and social difficulties not restricting out-migration to more prosper countries. One of the evidences of such institutional efficiency in Poland can be a dynamics of abortion percentage (abortion as percentage of pregnancies excluding fetal deaths/miscarriages). While in most of the examined transition countries abortion percentage grew after 1989, as one can see from Figure 5.4.5, in Poland, where this indicator was lowest in the region, a tendency was opposite.[5]

[5] Of course one can treat this as an example of institutional resistance. According to Wikipedia until 1932, abortion was banned in Poland without exceptions. In that year a new Penal Code legalized abortion strictly when there were medical reasons and, for the first time in Europe, when the pregnancy resulted from a criminal act. This law was in effect from 1932 to 1956. In 1956 the Sejm legalized abortion in cases where the woman was experiencing "difficult living conditions". After the fall of Communism, abortion debate erupted in Poland. Roman Catholic and Lutheran Churches, and right-wing politicians pressured the government to ban abortion except in cases where abortion was the only way to save the life of the

Figure 5.4.5. Abortion as percentage of pregnancies (excluding fetal deaths/miscarriages).



Surprisingly, deep econometric studies of population crisis conditioning factors in transition economies are not numerous. From these factors a fertility decline is investigated more often (see a survey provided by UNECE, 2000). The exception is Cornia and Paniccià (1998) who challenge the viewpoint that attributes the population crisis in transition economies to factors broadly unrelated to the economic and social difficulties experienced during the transition. They show that while important demographic changes occurred in the 1970s and 80s, in three-quarters of the cases examined the after 1989 shifts in nuptiality, fertility and mortality show large, growing and statistically significant variations from past trends. Authors find little or no evidence that these drastic variations are the result of shifts toward Western models of marriage or reproductive behavior. They instead explain these variations by negative shifts in the

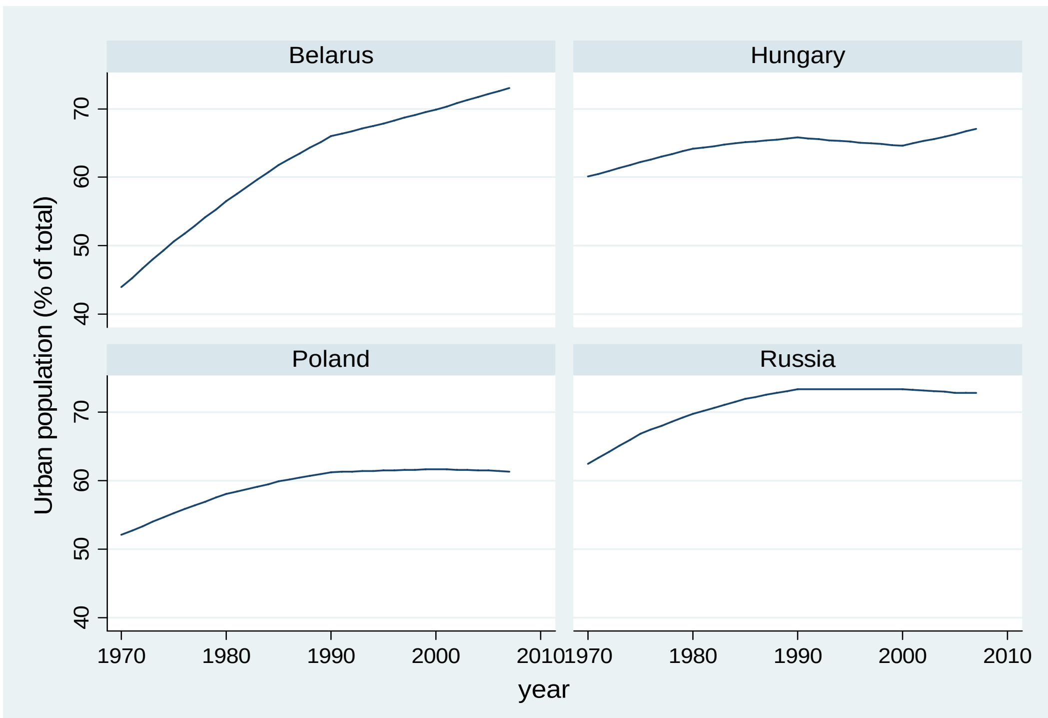
pregnant woman. Left-wing politicians and most liberals were opposed to this, and pressured the government to maintain the above mentioned 1956 legislation. The abortion law in Poland today was enacted in January 1993 as a compromise between both camps.
In 1997, parliament enacted a modification to the abortion bill which permitted the termination of pregnancy in cases of emotional or social distress, but this law was deemed unconstitutional by the Polish Constitutional Court. In December of that year the legal status of abortion in Poland was restored to that in 1993. Currently, Polish society is one of the most pro-life in Europe. In the poll European values in May 2005, 48% of Poles disagreed that a woman should be able to have an abortion if she doesn't want children. 47% were in favour of abortion. Out of the 10 polled countries, Poland was the only country where opposition to abortion was greater than support for abortion (http://en.wikipedia.org/wiki/Abortion_in_Poland).

economic circumstances of the marriageable population and of the families already formed, and in particular by the fall in real wages and rising cost of housing and other goods needed to establish and maintain a family. They are also due to the deterioration in and the modest impact of family policies on reproductive behaviour. In contrast, expectations about the economic outcomes of the current crisis appear to exert a sizeable influence on the decision to marry and, particularly, to have a child. UNECE (2000) results provide ample support for the hypothesis that the declines in household incomes have put downward pressure on fertility.

Looking for the explanation of cities population decline in the beginning of transition it is useful to bear in mind the urban sociologists' view that in the course of their evolution cities exploit not only a local site but a nodal geographical situation and develop as long as the networks they control are expanding (Pumain, 2010). Political and economic transition leads to multiple breaks in social and economic relationships. It is not unexpected then that even with large population increases in some cities due to nearby conflicts, the average metropolitan city in the former Soviet Union lost population between 1989 and 1997. For example, Moscow declined by 350,000 and St. Petersburg by more than 200,000 (Rowland 1998). At the same time over the period from the last Soviet census in January 1989 to the beginning of 1997, the net immigration to Russia offset the negative natural increase so that Russia's population increased over the period from 147,022 ths to 148,029 ths.

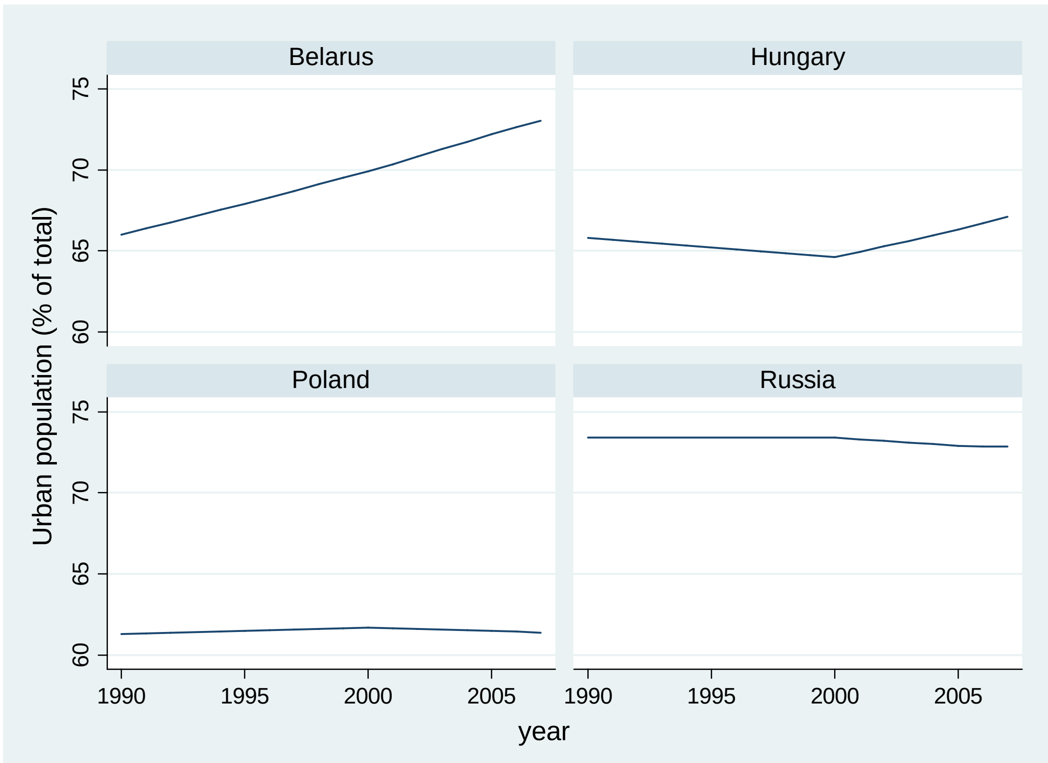
The explanation, at least partial, of this inverse population dynamics in the whole countries and theirs big cities could be behind the failure of industrialization policy. In contrast to nonsocialist economies, where urbanization is driven largely by market forces, socialist planners accelerated the process by moving people to cities more rapidly so that forced industrialization could generate faster economic development. From Chenery and Syrquin's (1986) results can be deduced that for a given level of per capita income, the share of the population in cities in the transition region was, on average, of the order of 12 percentage points higher than it was in comparator countries. Buckley and Mini (2000) stress that more important is that largely because the industrialization strategy failed, per capita income in 1990 was at least 40 percent lower than it was in countries that urbanized more spontaneously. After command system collapse peoples and firms start to take private decisions in an atmosphere of spatial competition. Unbalanced and undiversified industrial structure of socialist cities required deep structural changes and inter-industry reallocation of resources. Significant territorial adaptation and relocation of production factors among cities become a pressing task. With more freedom workers in over-industrialized cities, in words of Buckley and Mini (2000), can “vote with their feet” and move away from cities.

Figure 5.4.6. Urban population ratio in four transition countries (1970 - 2007)



In a historical perspective the patterns of urbanization for different countries are quite similar. However, evidently, the dynamics of urbanization is fastest in Belarus. It becomes even more obvious when we study 1990 – 2007 period (Figure 5.4.7). Recall, that it has appeared that the ergodic distribution for the country is more concentrated in the small and lower middle-size cities. The level of stability of ergodic distribution compared to the initial one for Belarus, Poland, and Russia is low, while it is relatively more stable for Hungary. For Belarus and Russia it appears that the ergodic distribution is more concentrated in the small and lower middle-size cities (1^{st} to 4^{th} classes), a result that reveals the existence of convergence towards smaller size populations. For Poland it appears that the ergodic distribution is more concentrated in the middle and big-size cities (5th to 7th classes).

Figure 5.4.7. Urban population ratio in four transition countries (1990 - 2007)



These differences in the long run patterns correlate more or less with the level of urbanization: it is relatively high for Belarus and Russia and in the long ran Makrov chains analysis predicts prevalence of small cities. Relatively low urbanization in Poland allows for use of potential of agglomeration economies and the dynamics of the "within" distribution confirms this by showing the picture of higher probability to move in the middle and big-size cities. The Hungarian distribution is between these extremes with more balanced distribution of cities between classes even in spite of some authors' observation that "formulation of a proper regional policy in Hungary remained incomplete" (Horváth, 1999). This is not the case of Poland with strong regional programs and of Russia and Belarus with relatively high and high centralization respectfully.

5.5 *Results concerning the factors driving the variation of the city size distribution*

To identify main drivers of city size distribution differences among examined countries and sequential policy implications we use panel data modeling to identify the determinants of the Pareto exponent variability. It is expected this should help us to understand better our results of studying cities distribution Pareto and non-Pareto behavior and their “within” movements.

In order to explain the differences in the city distributions, we will estimate a panel data fixed effects model. To ensure valid statistical inference we will employ cross-section dependence robust standard errors as explained in section 4.5.

Variables of the panel for Belarus, Hungary, Poland and Russia 1970-2007 annual data are presented in the Table 5.5.1.

Table 5.5.1. Description of the variables

pareto_cons	ζ_{it} consensus estimate of the Pareto exponent for the country i at time t
gdpa	Real 2005 GDP ($ths) per country area (sq km)
raila	Rail lines (total route-km) per country area (sq km)
mobpc	Mobile cellular subscriptions per 100 people
telpc	Telephone lines per 100 people
fri	Freedom index. It is an average of Political Rights and Civil Liberties indices measured on a one-to-seven scale, with one representing the highest degree of Freedom and seven the lowest.
prim1	Ratio of the lagest city population to the country population
prim5	Ratio of the 5 lagest city population to the country population
birthpc	Live births per 1000 people
abortion ratio	Abortions per 1000 live births
pop_log	Log of country population
gdppc_log	Log of country real 2005 GDP per capita ($)

Descriptive statistics for these variables are given in the Table 5.5.2.

Table 5.5.2. Summary statistics of the variables

Variable		Mean	Std. Dev.	Min	Max
gdpa	overall	387,1828	347,815	29,50352	1168,422
	between		367,2832	39,88817	790,977
	within		138,3204	61,55127	897,8645
raila	overall	4,822252	3,386811	0,462357	8,694053
	between		3,860494	0,494237	8,234675
	within		0,467875	3,114926	5,598575
telpc	overall	14,75578	10,61307	2,812716	37,75789
	between		1,458964	13,18703	16,05529
	within		10,53709	1,67023	36,57452
mobpc	overall	11,58132	27,41879	0	115,5061
	between		4,849984	5,671009	17,3746
	within		27,09302	-5,79328	116,4641
fri	overall	4,842105	2,112264	1	7
	between		1,467838	3,552632	6,368421
	within		1,68376	1,973684	7,289474
prim1	overall	0,109544	0,062161	0,040217	0,203554
	between		0,069467	0,043094	0,188427
	within		0,014861	0,05976	0,147687
prim5	overall	0,194024	0,080886	0,105446	0,340832
	between		0,088904	0,116721	0,282678
	within		0,023985	0,103625	0,252178
ab_ratio	overall	1033,031	721,8916	0,34	2541,2
	between		759,8259	149,9337	1922,903
	within		291,9902	28,72814	1651,328
birthpc	overall	13,34557	3,389471	8,134464	19,70818
	between		0,988537	12,34449	14,69424
	within		3,278834	7,74579	19,42145
pop_log	overall	17,11243	1,099001	16,01575	18,81603
	between		1,263957	16,09978	18,7726
	within		0,040979	16,98827	17,16126
gdppc_~g	overall	8,38544	0,459095	7,428048	9,298145
	between		0,45203	7,761562	8,843453
	within		0,237708	7,881959	9,01591

The *fixed effects model* allows the intercept to vary across countries, while keeping the slope coefficients the same for all 4 countries. The model can be made explicit for our application by inserting a 0-1 covariate for each of the countries except the one for which comparisons are to be made. The estimated equation is:

$$\zeta_{it} = \beta_1 + \beta_2 EcGeo_{it} + \beta_3 ICT_{it} + \beta_4 SocPolit_{it} + \beta_5 YEAR_t + \beta_6 CONTR_{it} + \varepsilon_{it} \qquad (1)$$

where ζ_{it} is the Pareto exponent, *EcGeo* is the vector of economic geography variables (real 2005 GDP ($ths) per country area (sq km), rail lines (total route-km) per country area (sq km)), ICT is the vector of *information and communication technologies (*mobile cellular subscriptions per 100 people, telephone lines per 100 people), SocPolit is a group of political and social variables (Freedom index defined as an average of Political Rights and Civil Liberties indices measured on a one-to-seven scale, with one representing the highest degree of Freedom and seven the lowest, Primacy index1 defined as a Ratio of the lagest city population to the country population, Primacy index1 defined as a Ratio of the 5 lagest city population to the country population, Abortions per 1000 live births). CONTROL is a set of variables controlling for the size of the country; here the control variables used are the log of the real 2005 GDP per capita in constant US dollars and the log of population.

Table 5.5.3 presents the results using the OLS estimate of the Pareto exponent as the dependent variable. Column (1) is the model without country controls. Both economic geography variables, real GDP per sq km of the country area and rail lines density, appear to facilitate the more even distribution of the cities. We cannot say the same about the influence of the information and communication technologies: proxy variable illustrating a popularity of mobile cellular services provided to be a factor explaining the bigger agglomerations development. Again primacy measured as a dominance of the 5 biggest cities has a negative effect on Pareto exponent thus contributing to less even development of urban systems.

Index of political freedom enters with the theoretically predicted sign but is not significant at 5% level. It is interesting to note that the sign of the coefficient which held such a sensitive variable as abortion ratio (illustrating abortions per 1000 live births) confirms its connection with uneven urbanization.

Table 5.5.3. Panel estimation of the model (dependent variable - pareto_cons)

Independent variable	(1)	(2)
gdpa	.00036626	.00011472
	(5.19) ***	(1.48)
raila	.06593139	.00897641
	(4.17) ***	(0.61)
telpc	.00108669	-.00468902
	(1.03)	(4.25) ***
mobpc	-.00079857	-.00153218
	(3.56) ***	(7.49) ***
fri	-.00590168	.0021019
	(1.08)	(0.46)
prim1	.86097608	1.3577834
	(0.45)	(0.86)
prim5	-3.012506	-3.7829106
	(2.61) *	(3.89) ***
abortion ratio	-.00004309	-2.226e-06
	(2.30)*	(0.13)
pop_log		-1.1784986
		(7.90) ***
gdppc_log		.13604305
		(3.97) ***
year	.0004134	.0100561
	(0.26)	(5.84) ***
Constant	.5110595	.84262033
	(0.17)	(0.32)
R-squared	0.7406	0.8289

t statistics in parentheses. * Significant at 5%; ** significant at 1%; *** significant at 0,1% level.

Including controls for country size (column (2)) shows that the results of the economic geography variables are not robust. The same is stressed by Soo (2005) in his analysis of 44 countries panel. This contrasts with the strong robustness of the information and communication technologies variables. The only robustly significant variable from the social and political group is the level of primacy of the 5 biggest cities, and this enters with the sign we would expect from theoretical reasoning. Thus, these results suggest that political factors play a more important role than economic geography variables in driving variation in the Pareto exponent across countries.

The signs of all significant variables remain unchanged in both equations. Intraclass correlation (rho) suggests that almost all the variation in Pareto exponent is related to inter countries differences (see Tables A.5.5.1-2 in the Appendix). The F tests indicate that there are significant individual (country level) effects implying that pooled OLS would be inappropriate. Nevertheless we have run OLS and can see that the fixed effects estimates of the panel are considerably lower than the OLS estimates, suggesting that the OLS estimates were inflated by unobserved heterogeneity. The Hausman test rejects the null hypothesis that the coefficients estimated by the efficient random effects estimator are the same as the ones estimated by the consistent fixed effects estimator.

Comparing our results to previous findings, one can see that our results are quite in line with findings of Soo (2005). At the same time, we have to some extent different results from those of Soo (2005) and Rosen and Resnick (1980), as they find that the Pareto exponent is positively related to total population. Our specification demonstrates larger R-squared compared to those of both Soo (2005) and Rosen and Resnick (1980) papers.

6 Concluding Remarks

This paper analyzed the dynamics of the city size distribution in CEE and CIS transition economies. Using a comprehensive unified database for CEE and CIS countries concerning city dynamics we tested the validity of Gibrat`s law employing panel unit root tests that takes into account the presence of cross-sectional dependence and Nadaraya-Watson non-parametrical kernel regression. We also constructed a consensus estimate of the Pareto exponent of the city distribution using various econometric methods. In order to test for non-Pareto behavior of the distribution when all the cities in a country are considered, we employed the Weber-Fechner law, the logarithmic hierarchy model, and the log-normal distribution. Not only we consider various distributions, but also study the "within distribution" dynamics by analyzing the individual cities relative positions and movement speeds in the overall distribution using a Markov chains methodology. In order to explain the differences in the city distributions and obtain valid statistical inference, we estimated, using cross-section dependence robust standard errors, a panel data fixed effects model to control for unobserved country specific determinants.

To test the fulfillment of the Gibrat`s law we explored the dynamics of city growth rates in twelve transition economies from the former communist block, namely Russia, Ukraine, Poland, Romania, Belarus, Bulgaria, Hungary, Czech Republic, Slovak Republic, Estonia, Latvia and Lithuania. We employed both detailed city data in the period 2000-2009 for Poland, Belarus and Latvia, as well as data on cities over 100,000 inhabitants in the period 1970-2007 for all the twelve countries. Regarding the detailed city data, the estimates of the pooled model, using both parametric and non-parametric methods, provide evidence for the rejection of Gibrat`s law in the three analyzed countries. On the other hand, when accounting for city specific effects, there is support for the acceptance of the law of proportional effect, with cities seemingly growing independent of their size. The latter evidence is also confirmed by the panel unit root tests. However, in the case of Belarus, as indicated by the non-parametric methods and confirmed by a deeper parametric analysis, there is a significant difference between the behavior of small and large cities, with the growth of large ones having a significant dependence on size. Overall, in the period 2000-2009 there is strong evidence that Gibrat`s law holds for Latvia and Poland. However, at least in the short run, a divergence pattern was detected in the case of Belarus. The other major contribution resides in the analysis conducted for cities over 100,000 inhabitants using yearly data for the period 1970-2007. Two main problems had to be addressed, respectively the existence of a potential break in the deterministic component of the growth rates of the cities in the former communist block, and missing observations given limited availability of data. After the influence of the change in the deterministic component is accounted for, there is strong support for the validity of Gibrat`s law in Poland, Romania, Belarus, Bulgaria, Former Czechoslovakia (Czech Republic, Slovak Republic), and the Baltic States (Estonia, Latvia and Lithuania), with weaker support for Hungary, Russia and Ukraine. In order to ensure robustness, the analysis has also been conducted using five years averages, with the results largely confirming the findings using yearly data. Overall, the findings indicate that there is strong support for accepting Gibrat`s law in Poland, Romania, Belarus, Bulgaria, Hungary, Former Czechoslovakia (Czech Republic, Slovak Republic), and the Baltic States (Estonia, Latvia and Lithuania).

Regarding the city size distribution, for the large majority of countries and time periods the estimated Pareto coefficient is higher than one. However, one can not reject that the Pareto exponent is significantly different from one, and therefore it seems that the Zipf Law holds. This is in line with other studies in the literature that obtained favorable evidence of Zipf’s Law in the upper-tail distribution of cities. The distribution of the size of the largest cities of Russia, Belarus, Central Asia, Caucasus, Poland and Hungary is consistent with Zipf's law. This is natural, as if, there are mega-cities whose size is very large compared with the size of other cities, Zipf's law is performed automatically. It all depends on the choice of the truncation of the tail distribution; to measure the tail indices of the distributions are approximately equal to one. These mega-cities of Russia is Moscow and St. Petersburg, in Belarus - Minsk, in Central Asia - Tashkent, in the Caucasus - Baku.

The distribution of the size of the size (all) cities of Russia, Belarus, Central Asia, Caucasus, Poland and Hungary satisfies the law of Weber-Fechner except the largest mega-cities. This fact is interesting because in contrast to Zipf's law Weber-Fechner law holds for all localities, not only for the largest cities. On the contrary, most large cities do not obey the Weber-Fechner. Changing the model of Weber-Fechner allows us to study the influence of time, as well as various political factors (shock) on the rate of urban development.

The Great October Revolution and World War II led to an increase in Russian cities due to influx of rural population in the city. When Stalin began forced urbanization, people from villages in the 30 th, 40 th, 50 th years, went into the city. The collapse of the USSR led to relative reduction cities of Central Asia and Caucasus as a result of relocation of non-indigenous population in rural areas of Russia. The collapse of the USSR at the rate of urban growth in the Belarus statistically significant effects are not influence. Apparently, Belarus has not experienced the shocking collapse of the lifestyle as a result of the collapse of the Soviet Union, as other CIS countries.

The First World War did not have a statistically significant impact on the development of towns in Hungary, the Second World War gave the effect of reducing the overall scale of cities and growth of middle-sized and small cities. Post-Communist regime for the overall scale of the cities were not affected, but gave the effect of reducing the rate of urban growth. The Distribution of cities in Russia, Belarus, Central Asia, Caucasus, Poland and Hungary is best described by models based on the hierarchy of the logarithms of their sizes. This phenomenon needs to be sociological (and economic) explanation for the analogy explanation made Gabaix for Zipf's law in (Gabaix, X. (1999), "Zipf's Law for cities: an Explanation", *Quarterly Journal of Economics.*).

To analyze the "within distribution" movement of individual cities, we consider time dynamics of urban systems of four countries: Poland (890 cities for period 1961 - 2004), Belarus (207 cities for period 1970 - 2009), Hungary (237 cities for period 1880 - 2001), Russia (479 cities for period 1897 - 2002) and presence of spatial autocorrelation of Belarusian cities.

The Markov chains analysis shows a low interclass mobility, i.e., a high persistence of cities to stay in their own class over the whole period. In general, the largest and smallest cities display higher persistence than the medium-sized cities, which have more probability of moving to smaller categories. In general terms, movements up are slower than movements down, especially for high-size classes.

Comparing ergodic distributions and mean first passage time matrices for Belarus and Poland we may conclude that in the future 56% of Belarusian cities will be smaller than 10% of the Belarusian average and passage of cities from higher classes to lower is more probable. Future distribution of Polish cities is an opposite to Belarusian pattern and tends to big cities (up to 64% of all Polish cities will be greater than the Polish average city size). Russian cities will evolve mostly similar as Belarusian pattern, but there is a difference concerning 7 class. Russian 7 class will be greater than Belarusian one.

The difference in the models of urban system development and the forms of cities' convergence for Belarus on the one part and Poland on the other becomes obvious after comparison of initial versus ergodic distribution patterns matching. Concentration of the frequencies in the class of small cities is registered for Belarus and Russia, while one can see the opposite for Poland. The behavior of Hungarian initial and ergodic distributions are more stable and form-preserving among all others and look like Gaussian distributions with maximums at medium classes: 5th class for initial distribution and 4th class for ergodic one. It shows a shift towards one class smaller cities and increase of the distribution variance.

Spatial analysis of Belarusian cities underlines existence of divergence of the urban system in space, not only in time. It may be a consequence of a significant role of the state in the economy and concentration of resources in big cities. As a result we have a designed hierarchy of cities or at least the hierarchy which is shaped for the most part not by market forces but rather by visible hand of the state. This conclusion is supported by our results which indicate no strong support for Gibrat`s model of urban system development in the case of Belarus. Revealed doubts in cities proportionate growth in Belarus coincides with presence of spatial autocorrelation in urban systems. Some additional comparisons of the pre and post 1989 development of the examined countries with detailed data show that in a historical perspective the patterns of urbanization for them are quite similar. However, after 1989 the picture is quite different: the dynamics of urbanization is significant only in Belarus. Mentioned above differences in the long run patterns of urban system distributions correlate with the level of urbanization: it is relatively high for Belarus and Russia and Makrov's chains analysis predicts prevalence of small cities in the future. Rather low urbanization level in Poland allows for use of agglomeration economies and the dynamics of the "within" distribution confirms this by showing the picture of higher probability to move in the middle and big-size cities. The Hungarian distribution is between these extremes with more balanced distribution of cities between classes even in spite of an expert's opinion that proper regional policy in Hungary remained incomplete. This is not the case of Poland with strong regional programs and of Russia and Belarus with relatively high and high centralization respectfully. This gives us the opportunity to propose that market forces via mechanism of spatial competition lead to more even distribution of population then development and implementation of intentional regional policies.

The main value added of our research is looking at the cities distribution from different perspectives (different theoretical and empirical laws of distributions, within dynamics). To answer the question about the sources of cities distribution differences among countries we use panel data techniques. It is expected this should help us to understand our results of Pareto and non-Pareto behavior of cities distributions and their within movements. Urban and regional policy implications could be based on derived conclusions.

Fixed effects model estimations controlling for country size show that economic geography variables are not robust what is in agreement with Soo (2005). This contrasts with the strong robustness of the information and communication technologies variables. The only robustly significant variable from the social and political group is the level of primacy of the 5 biggest cities which enters with the negative sign. This result confirms that political factors play a more important role than economic geography variables in driving variation in the Pareto exponent across countries (assuming this variable is a good proxy for the level of centralization and state intervention). The sign of the primacy variable coefficient indicates that the lower political intervention means the more even population distribution. Our general conclusion thus is that political intervention with significant probability takes the form of the expansion of the largest cities and the size distribution becomes more unequal.

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Appendix

Table A.5.1.1 *Summary statistics of the data employed in testing the validity of Gibrat`s Law*

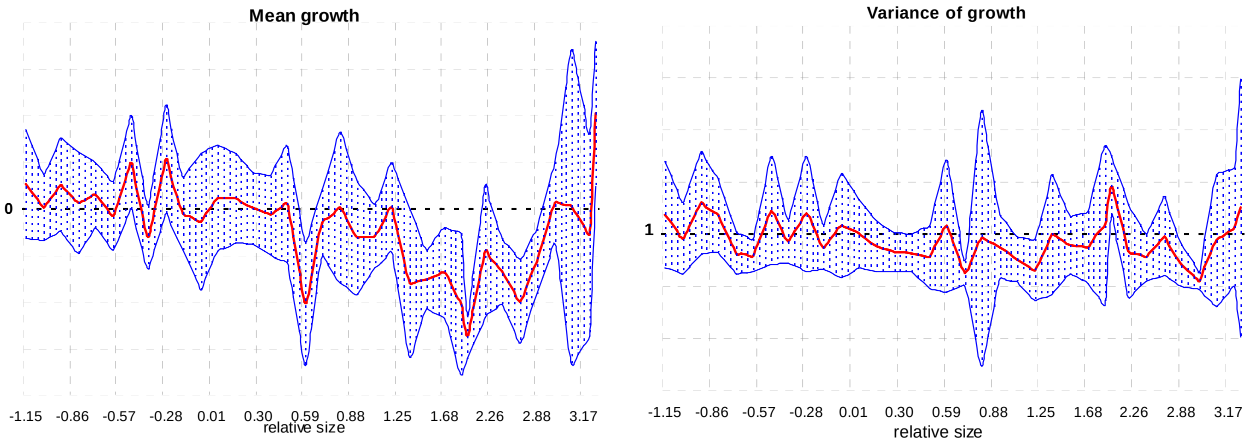
	Data on cities over 100,000 inhabitants									Detailed city data			
	Russia	Ukraine	Poland	Romania	Belarus	Bulgaria	Hungary	Fr. Czechosl.	Baltic States		Poland	Belarus	Latvia
no. obs.	3644	741	995	554	351	226	313	197	260	no. obs.	2000	500	300
period	1970 - 2007	1970 - 2007	1970 - 2007	1970 - 2007	1970 - 2007	1970 - 2007	1970 - 2007	1970 - 2007	1970 - 2007	period	2000-2009	2000-2009	2000-2009
T dim.	24	17	27	26	27	28	38	25	32	T dim.	10	10	10
CS dim.	164	51	43	26	15	11	9	10	9	CS dim.	200	50	30
Average	416,797	401,355	285,662	281,715	321,515	297,009	370,851	360,179	331,561	Average	90,701	120,767	48,314
Std. dev.	816,582	437,791	282,120	368,143	378,803	300,340	594,286	330,581	235,663	Std. dev.	159,557	255,850	130,224
Min	90,000	100,000	96,648	99,494	91,300	96,099	100,100	94,436	100,431	Min	21,710	15,100	7,943
Max	10,456,490	2,676,789	1,704,717	2,127,194	1,797,500	1,155,403	2,116,548	1,216,568	917,000	Max	1,709,781	1,829,100	766,381

Table A.2 *Growth regression results using detailed city data in Belarus for the period 2000-2009*

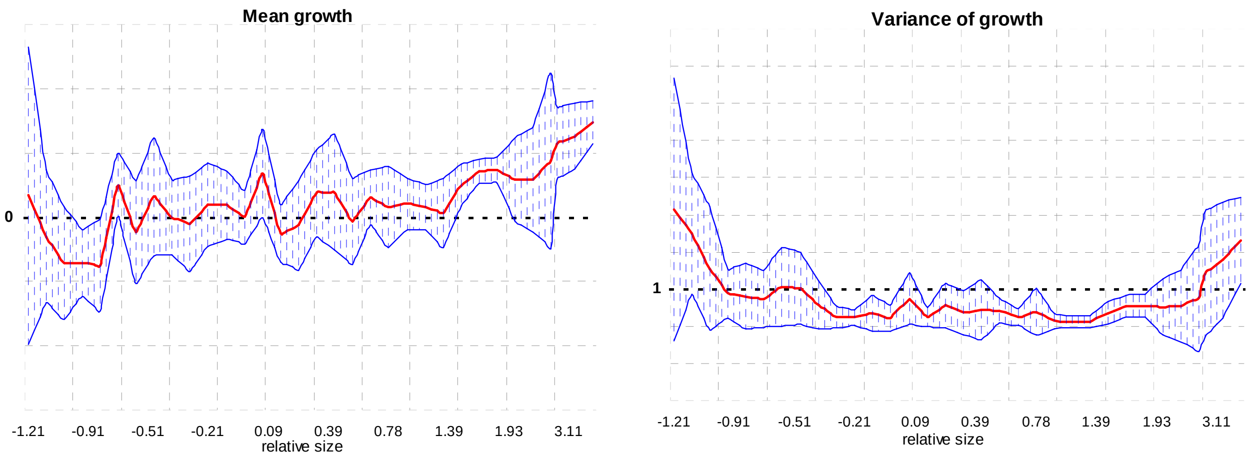
	all sample	large cities	medium cities	small cities
ln(Size)	0.0015	0.0030	0.0006	0.0085
	[0.0008]	[0.0007]	[0.0008]	[0.0064]
	(0.0461)	(0.0043)	(0.4438)	(0.2062)
d_medium	-0.0035			
	[0.0016]			
	(0.0287)			
d_small	-0.0050			
	[0.0014]			
	(0.0011)			
HWH		7.8100	0.7600	1.7900
		(0.0267)	(0.3900)	(0.2028)

d_medium is a dummy variable controlling for medium cities and d_small a dummy variable contolling for small ones; Driscoll - Kraay robust standard errors are reported in squared parentheses; p-values are reported in round parentheses; HWH is the modified Hausman (1978) test.

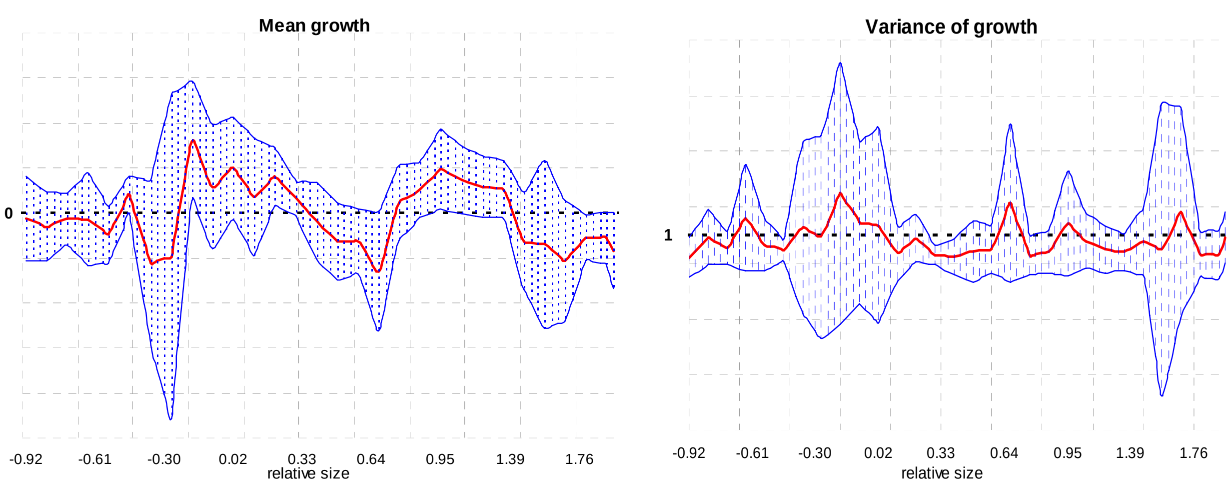
Figure A.5.1.1. *Non-parametric estimation using detailed city data in Poland, Belarus and Latvia for the period 2000-2009*



a. Poland

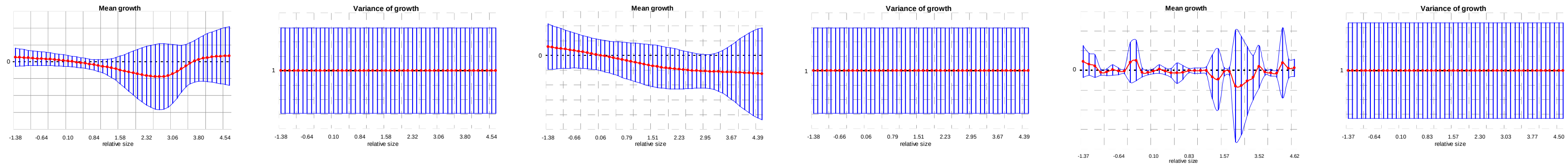


b. Belarus



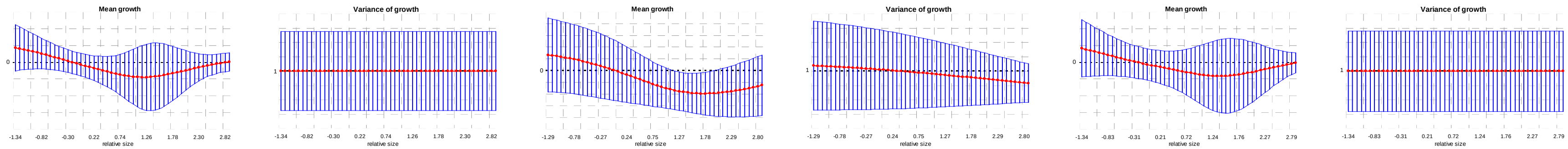
c. Latvia

Figure A.5.1.2. *Non-parametric estimation for cities over 100,000 inhabitants for the period 1970-2007*



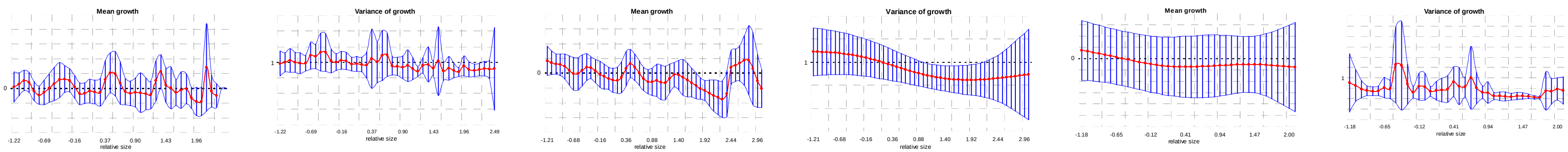
all sample before 1989 after 1989

a. Russia



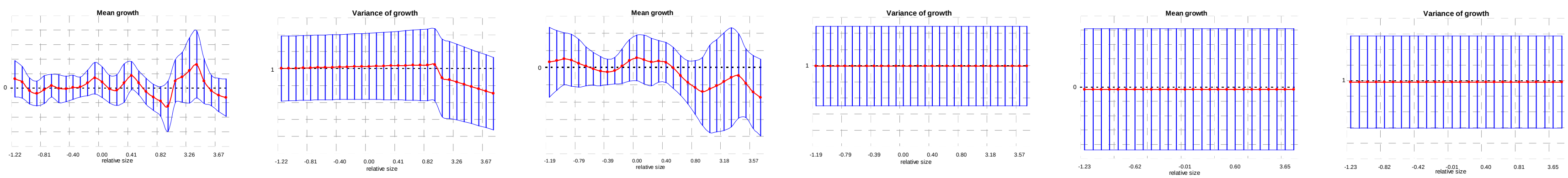
all sample before 1989 after 1989

b. Ukraine



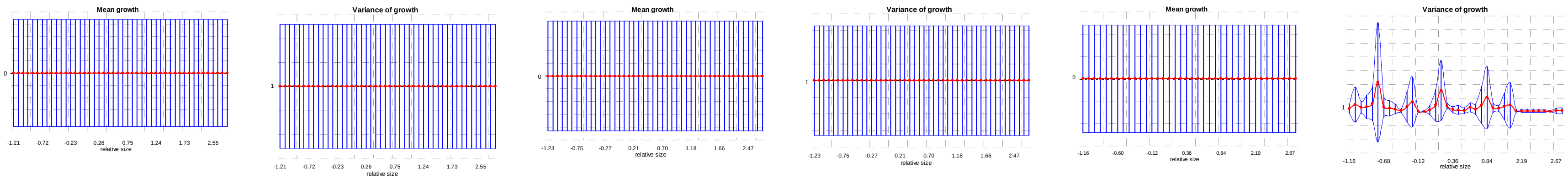
all sample before 1989 after 1989

c. Poland



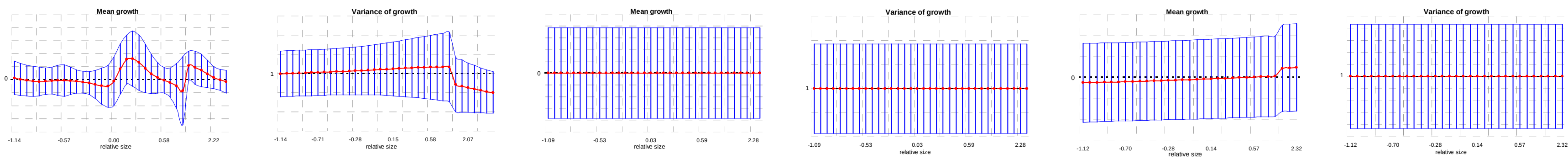
all sample before 1989 after 1989

d. Romania



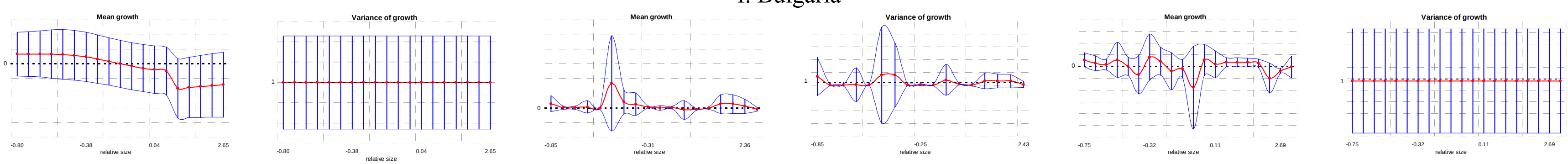
all sample before 1989 after 1989

e. Belarus



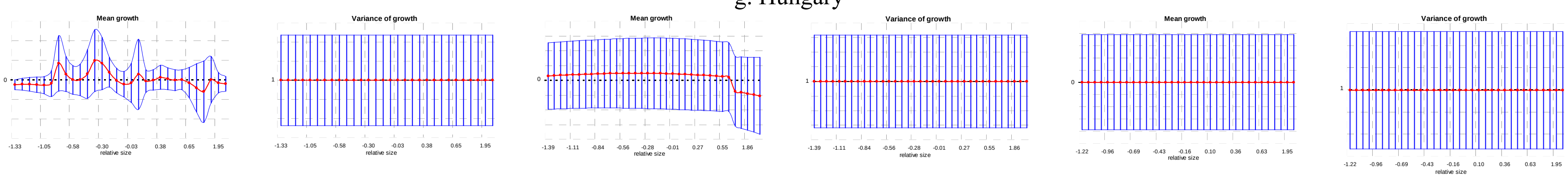
all sample before 1989 after 1989

f. Bulgaria



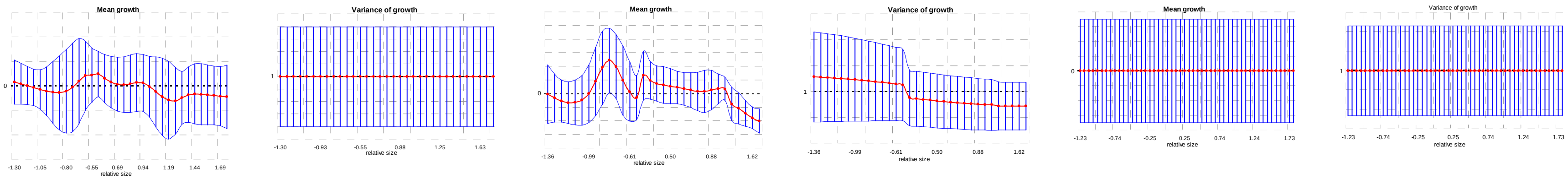
all sample before 1989 after 1989

g. Hungary



all sample before 1989 after 1989

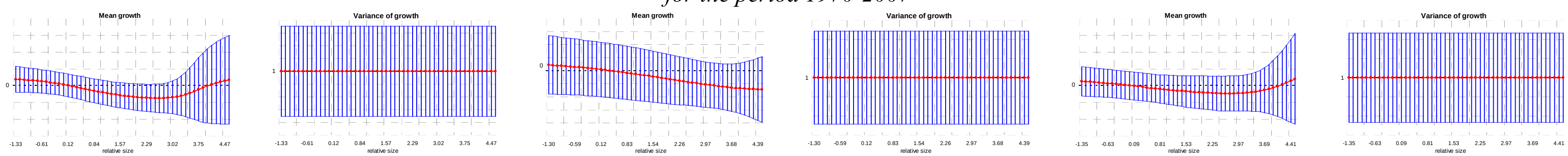
h. Former Czechoslovakia



all sample before 1989 after 1989

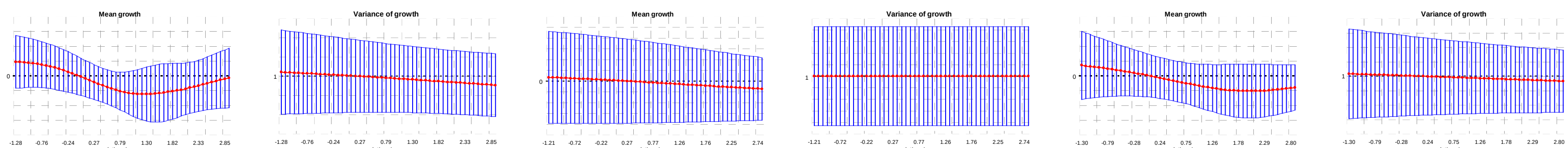
i. Baltic States

Figure A.5.1.3. *Non-parametric estimation for cities over 100,000 inhabitants using five years averages for the period 1970-2007*



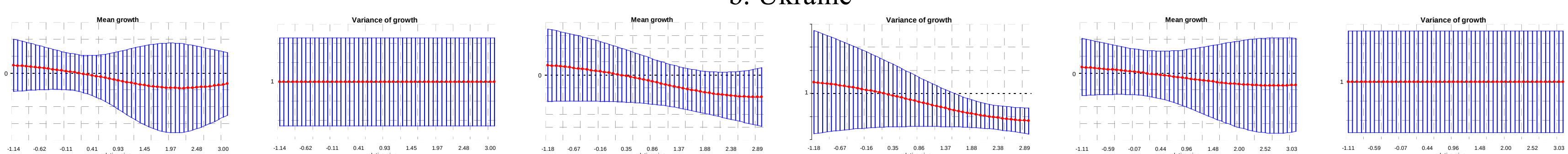
all sample before 1989 after 1989

a. Russia



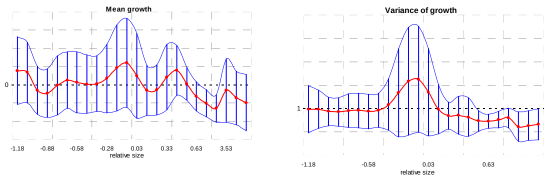
all sample before 1989 after 1989

b. Ukraine

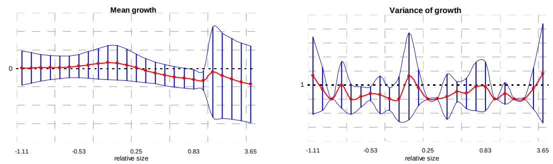


all sample before 1989 after 1989

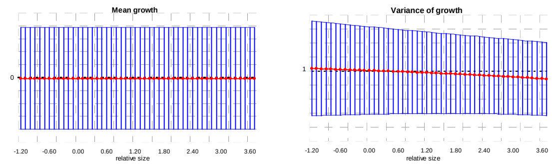
c. Poland



all sample

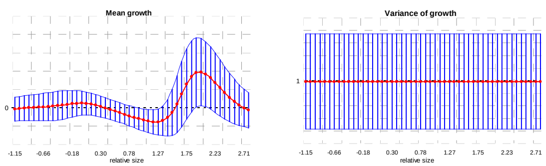


before 1989

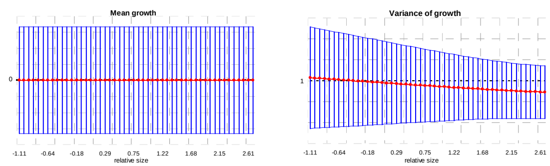


after 1989

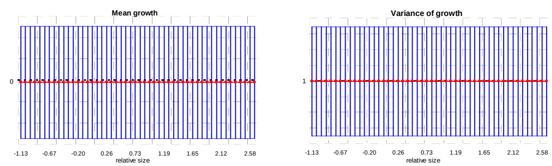
d. Romania



all sample

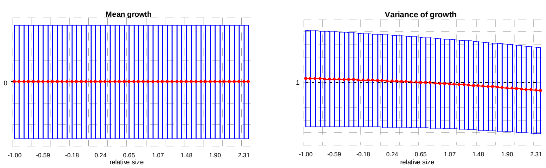


before 1989

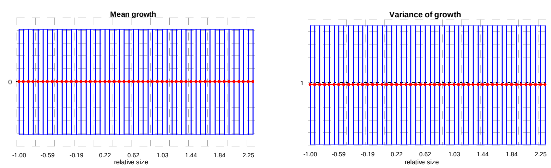


after 1989

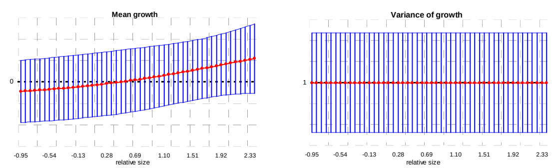
e. Belarus



all sample

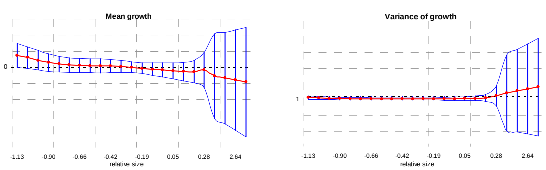


before 1989

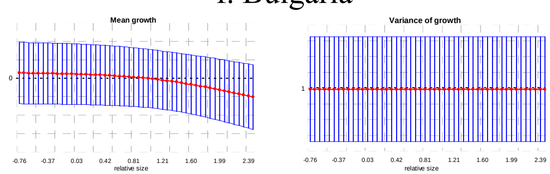


after 1989

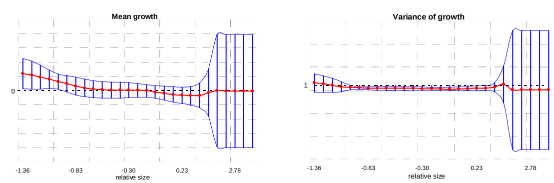
f. Bulgaria



all sample

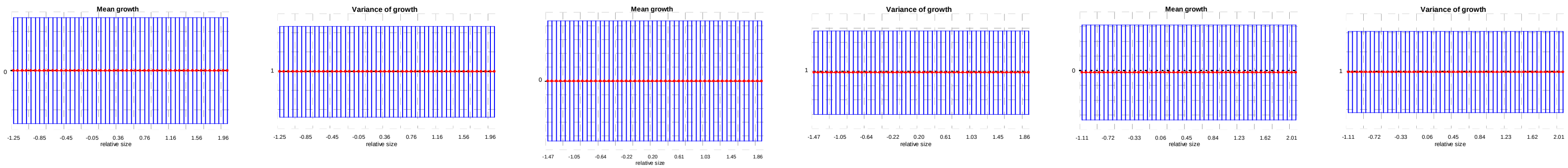


before 1989



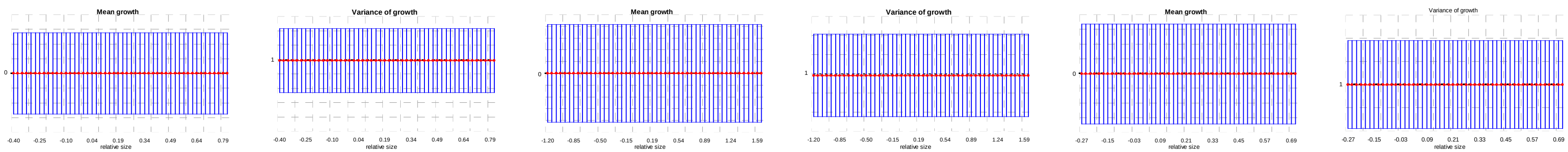
after 1989

g. Hungary



all sample before 1989 after 1989

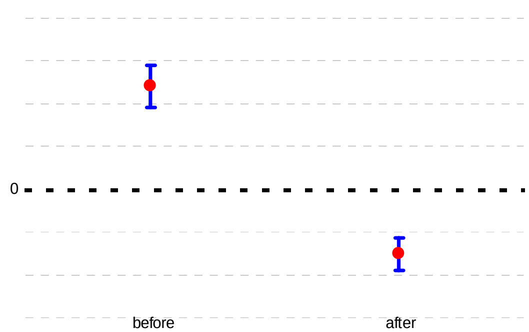
h. Former Czechoslovakia



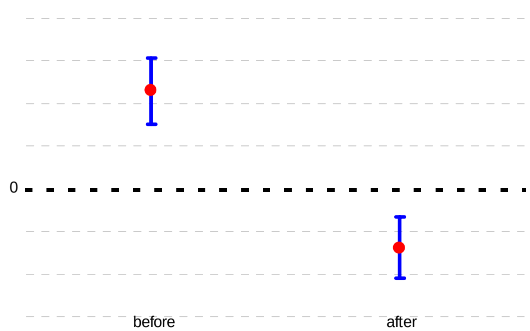
all sample before 1989 after 1989

i. Baltic States

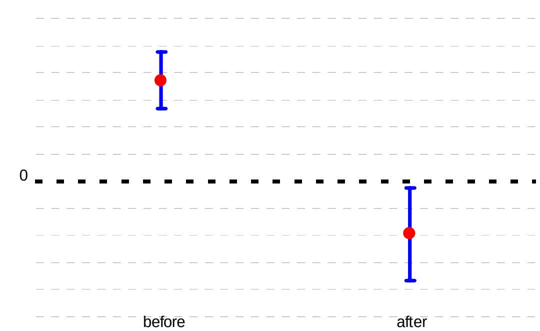
Figure A.5.1.4. *The non-parametrical estimates of the potential shift in the deterministic component of growth rates using five years averages*



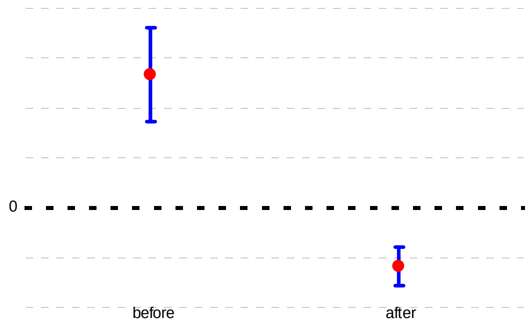
a. Russia



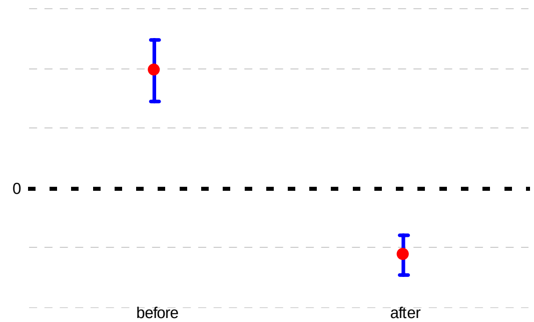
b. Ukraine



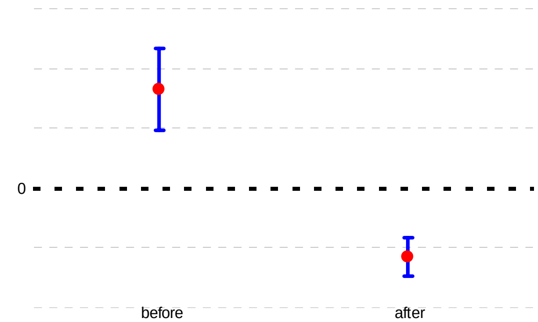
c. Poland



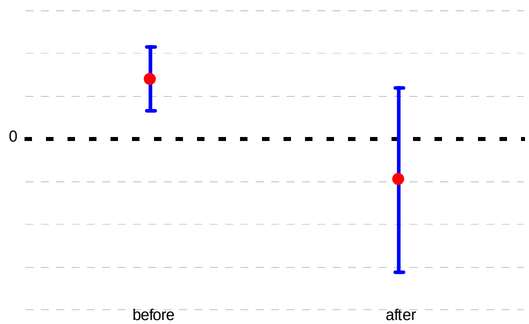
d. Romania



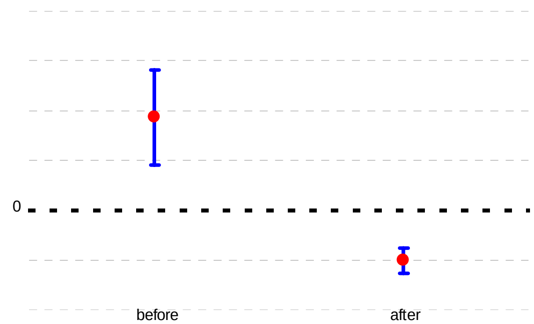
e. Belarus



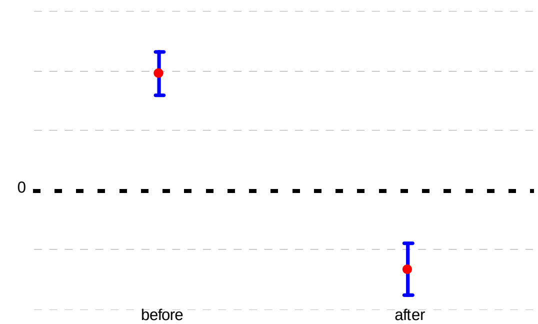
f. Bulgaria



g. Hungary



h. Former Czechoslovakia



i. Baltic States

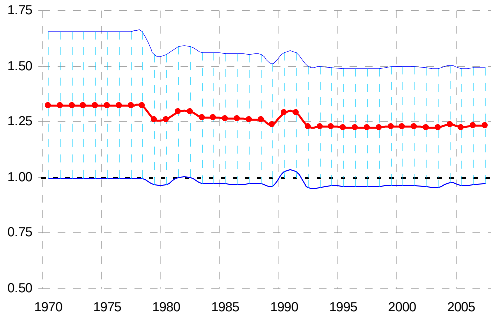
Table A.5.2.1 *The estimates for the Pareto coefficient of city size distribution in CEE and CIS countries*

Year	Poland		Romania		Hungary		Bulgaria		Belarus		Former Yugoslavia		Former Czechoslovakia		Baltic States		Ukraine		Russia	
	Reg.	MLE	Reg.	MLE	Reg.	MLE	Reg.	MLE	Reg.	MLE	Reg.	MLE	Reg.	MLE	Reg.	MLE	Reg.	MLE	Reg.	MLE
1970	1.421	1.199	1.275	2.066	0.743	1.336	1.168	1.466	1.399	1.479	1.271	1.243	1.157	1.413	1.107	0.996	1.168	1.021	1.325	1.066
	[0.419]	[0.25]	[0.499]	[0.573]	[0.428]	[0.545]	[0.674]	[0.598]	[0.659]	[0.492]	[0.635]	[0.439]	[0.668]	[0.576]	[0.639]	[0.406]	[0.264]	[0.163]	[0.168]	[0.095]
1971	1.451	1.215	1.300	1.664	0.752	1.333	1.169	1.466	1.399	1.479	1.360	1.650	1.172	1.433	1.104	0.988	1.168	1.021	1.325	1.066
	[0.418]	[0.248]	[0.491]	[0.444]	[0.434]	[0.544]	[0.675]	[0.598]	[0.659]	[0.492]	[0.641]	[0.549]	[0.676]	[0.584]	[0.637]	[0.403]	[0.264]	[0.163]	[0.168]	[0.095]
1972	1.451	1.215	1.305	1.668	0.757	1.344	1.190	1.488	1.399	1.479	1.360	1.650	1.190	1.426	1.114	0.988	1.168	1.021	1.325	1.066
	[0.418]	[0.248]	[0.493]	[0.445]	[0.436]	[0.548]	[0.635]	[0.562]	[0.659]	[0.492]	[0.641]	[0.549]	[0.687]	[0.582]	[0.643]	[0.403]	[0.264]	[0.163]	[0.168]	[0.095]
1973	1.421	1.170	1.330	1.650	0.764	1.298	1.193	1.518	1.399	1.479	1.360	1.650	1.190	1.426	1.076	1.107	1.168	1.021	1.325	1.066
	[0.401]	[0.233]	[0.485]	[0.426]	[0.44]	[0.529]	[0.637]	[0.573]	[0.659]	[0.492]	[0.641]	[0.549]	[0.687]	[0.582]	[0.575]	[0.418]	[0.264]	[0.163]	[0.168]	[0.095]
1974	1.421	1.193	1.334	1.678	0.768	1.313	1.196	1.535	1.399	1.479	1.358	1.658	1.213	1.416	1.086	1.128	1.168	1.021	1.325	1.066
	[0.401]	[0.238]	[0.486]	[0.433]	[0.443]	[0.535]	[0.639]	[0.58]	[0.659]	[0.492]	[0.64]	[0.552]	[0.7]	[0.578]	[0.58]	[0.426]	[0.264]	[0.163]	[0.168]	[0.095]
1975	1.421	1.193	1.334	1.678	0.773	1.326	1.185	1.483	1.399	1.479	1.358	1.658	1.213	1.416	1.144	1.188	1.168	1.021	1.325	1.066
	[0.401]	[0.238]	[0.486]	[0.433]	[0.446]	[0.541]	[0.633]	[0.56]	[0.659]	[0.492]	[0.64]	[0.552]	[0.7]	[0.578]	[0.611]	[0.448]	[0.264]	[0.163]	[0.168]	[0.095]
1976	1.413	1.173	1.334	1.678	0.777	1.339	1.174	1.435	1.399	1.479	1.358	1.658	1.235	1.375	1.082	1.128	1.168	1.021	1.325	1.066
	[0.377]	[0.221]	[0.486]	[0.433]	[0.448]	[0.546]	[0.627]	[0.542]	[0.659]	[0.492]	[0.64]	[0.552]	[0.712]	[0.561]	[0.578]	[0.426]	[0.264]	[0.163]	[0.168]	[0.095]
1977	1.394	1.300	1.352	1.460	0.820	1.233	1.185	1.483	1.399	1.479	1.358	1.658	1.202	1.360	1.086	1.125	1.168	1.021	1.325	1.066
	[0.343]	[0.226]	[0.45]	[0.344]	[0.438]	[0.465]	[0.633]	[0.56]	[0.659]	[0.492]	[0.64]	[0.552]	[0.693]	[0.555]	[0.58]	[0.425]	[0.264]	[0.163]	[0.168]	[0.095]
1978	1.396	1.301	1.378	1.431	0.866	1.428	1.183	1.438	1.399	1.479	1.358	1.658	1.202	1.360	1.070	1.134	1.168	1.021	1.325	1.066
	[0.343]	[0.226]	[0.447]	[0.328]	[0.433]	[0.504]	[0.632]	[0.543]	[0.659]	[0.492]	[0.64]	[0.552]	[0.693]	[0.555]	[0.534]	[0.4]	[0.264]	[0.163]	[0.168]	[0.095]
1979	1.396	1.301	1.378	1.431	0.866	1.430	1.183	1.438	1.256	1.360	1.358	1.658	1.097	0.949	1.062	1.133	1.165	1.032	1.260	1.030
	[0.343]	[0.226]	[0.447]	[0.328]	[0.432]	[0.505]	[0.632]	[0.543]	[0.561]	[0.43]	[0.64]	[0.552]	[0.586]	[0.358]	[0.531]	[0.4]	[0.254]	[0.159]	[0.15]	[0.086]
1980	1.407	1.335	1.378	1.431	0.870	1.455	1.187	1.505	1.256	1.360	1.358	1.658	1.106	0.951	1.063	1.135	1.165	1.032	1.260	1.030
	[0.327]	[0.219]	[0.447]	[0.328]	[0.434]	[0.514]	[0.634]	[0.568]	[0.561]	[0.43]	[0.64]	[0.552]	[0.591]	[0.359]	[0.531]	[0.401]	[0.254]	[0.159]	[0.15]	[0.086]
1981	1.420	1.344	1.418	1.528	0.873	1.469	1.187	1.505	1.236	1.318	1.256	1.405	1.106	0.951	1.067	1.126	1.232	1.010	1.295	1.046
	[0.33]	[0.22]	[0.46]	[0.35]	[0.436]	[0.519]	[0.634]	[0.568]	[0.552]	[0.416]	[0.474]	[0.375]	[0.591]	[0.359]	[0.533]	[0.398]	[0.259]	[0.15]	[0.15]	[0.085]
1982	1.427	1.315	1.431	1.229	0.873	1.469	1.183	1.229	1.236	1.318	1.284	1.443	1.110	0.940	1.064	1.120	1.232	1.010	1.295	1.046
	[0.327]	[0.213]	[0.452]	[0.274]	[0.436]	[0.519]	[0.591]	[0.434]	[0.552]	[0.416]	[0.485]	[0.385]	[0.593]	[0.355]	[0.531]	[0.395]	[0.259]	[0.15]	[0.15]	[0.085]
1983	1.431	1.324	1.431	1.229	0.879	1.478	1.183	1.229	1.209	1.179	1.284	1.443	1.110	0.938	1.066	1.113	1.183	0.964	1.267	1.064
	[0.328]	[0.214]	[0.452]	[0.274]	[0.439]	[0.522]	[0.591]	[0.434]	[0.515]	[0.355]	[0.485]	[0.385]	[0.593]	[0.354]	[0.532]	[0.393]	[0.246]	[0.142]	[0.15]	[0.089]
1984	1.428	1.316	1.416	1.316	0.916	1.500	1.228	1.514	1.209	1.179	1.256	1.405	1.113	0.943	1.066	1.113	1.183	0.964	1.267	1.064

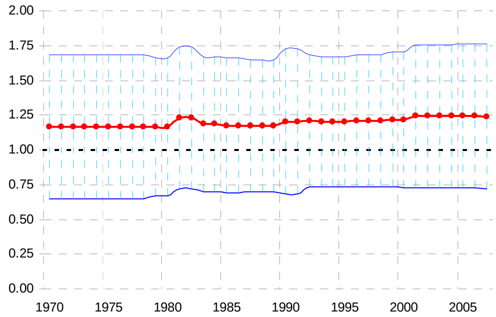
	[0.323]	[0.21]	[0.437]	[0.287]	[0.431]	[0.499]	[0.548]	[0.478]	[0.515]	[0.355]	[0.474]	[0.375]	[0.594]	[0.356]	[0.532]	[0.393]	[0.246]	[0.142]	[0.15]	[0.089]
1985	1.434	1.308	1.424	1.316	0.918	1.498	1.221	1.485	1.266	1.104	1.256	1.405	1.073	1.028	1.064	1.103	1.175	0.953	1.263	1.050
	[0.324]	[0.209]	[0.439]	[0.287]	[0.432]	[0.499]	[0.546]	[0.469]	[0.539]	[0.332]	[0.474]	[0.375]	[0.536]	[0.363]	[0.532]	[0.39]	[0.247]	[0.141]	[0.149]	[0.087]
1986	1.436	1.330	1.416	1.316	0.919	1.500	1.226	1.499	1.207	1.218	1.256	1.405	1.073	1.028	1.062	1.090	1.175	0.953	1.263	1.053
	[0.321]	[0.21]	[0.437]	[0.287]	[0.433]	[0.499]	[0.548]	[0.473]	[0.514]	[0.367]	[0.474]	[0.375]	[0.536]	[0.363]	[0.531]	[0.385]	[0.247]	[0.141]	[0.149]	[0.088]
1987	1.441	1.330	1.424	1.316	0.920	1.506	1.221	1.485	1.209	1.221	1.256	1.405	1.082	1.250	1.065	1.089	1.174	0.918	1.261	1.011
	[0.322]	[0.21]	[0.439]	[0.287]	[0.433]	[0.502]	[0.546]	[0.469]	[0.515]	[0.368]	[0.474]	[0.375]	[0.483]	[0.395]	[0.532]	[0.384]	[0.242]	[0.133]	[0.147]	[0.083]
1988	1.437	1.331	1.424	1.316	0.920	1.523	1.226	1.499	1.209	1.221	1.256	1.405	1.081	1.246	1.065	1.089	1.174	0.918	1.261	1.011
	[0.317]	[0.207]	[0.439]	[0.287]	[0.433]	[0.507]	[0.548]	[0.473]	[0.515]	[0.368]	[0.474]	[0.375]	[0.483]	[0.394]	[0.532]	[0.384]	[0.242]	[0.133]	[0.147]	[0.083]
1989	1.437	1.331	1.387	1.330	0.920	1.523	1.248	1.546	1.209	1.221	1.256	1.405	1.081	1.246	1.065	1.089	1.174	0.918	1.136	0.379
	[0.317]	[0.207]	[0.418]	[0.283]	[0.433]	[0.507]	[0.558]	[0.488]	[0.515]	[0.368]	[0.474]	[0.375]	[0.483]	[0.394]	[0.532]	[0.384]	[0.242]	[0.133]	[0.124]	[0.029]
1990	1.341	1.280	1.351	1.208	0.923	1.561	1.251	1.550	1.183	1.106	1.256	1.405	1.079	1.245	1.102	1.330	1.205	0.868	1.291	1.111
	[0.289]	[0.195]	[0.417]	[0.263]	[0.435]	[0.52]	[0.559]	[0.49]	[0.482]	[0.319]	[0.474]	[0.375]	[0.482]	[0.393]	[0.492]	[0.42]	[0.266]	[0.135]	[0.136]	[0.083]
1991	1.439	1.404	1.351	1.208	0.923	1.561	1.251	1.550	1.183	1.106	1.291	1.473	1.080	1.247	1.110	1.320	1.205	0.868	1.291	1.111
	[0.31]	[0.214]	[0.417]	[0.263]	[0.435]	[0.52]	[0.559]	[0.49]	[0.482]	[0.319]	[0.418]	[0.337]	[0.482]	[0.394]	[0.496]	[0.417]	[0.266]	[0.135]	[0.136]	[0.083]
1992	1.439	1.404	1.443	1.400	0.927	1.591	1.204	1.416	1.183	1.115	1.354	1.580	1.077	1.254	1.131	1.401	1.210	1.018	1.190	0.472
	[0.31]	[0.214]	[0.4]	[0.274]	[0.436]	[0.53]	[0.567]	[0.472]	[0.483]	[0.321]	[0.417]	[0.344]	[0.481]	[0.396]	[0.482]	[0.422]	[0.241]	[0.143]	[0.134]	[0.037]
1993	1.442	1.385	1.402	1.339	0.929	1.600	1.214	1.403	1.175	1.135	1.361	1.523	1.077	1.254	1.134	1.366	1.204	0.979	1.190	0.472
	[0.314]	[0.213]	[0.396]	[0.267]	[0.438]	[0.533]	[0.543]	[0.443]	[0.46]	[0.314]	[0.42]	[0.332]	[0.481]	[0.396]	[0.483]	[0.411]	[0.238]	[0.137]	[0.134]	[0.037]
1994	1.439	1.404	1.451	1.333	0.931	1.634	1.213	1.425	1.175	1.135	1.341	1.509	1.055	1.133	1.135	1.353	1.204	0.979	1.229	1.063
	[0.31]	[0.214]	[0.41]	[0.266]	[0.439]	[0.544]	[0.542]	[0.45]	[0.46]	[0.314]	[0.413]	[0.329]	[0.497]	[0.377]	[0.484]	[0.407]	[0.238]	[0.137]	[0.134]	[0.082]
1995	1.442	1.385	1.455	1.321	0.933	1.679	1.214	1.403	1.175	1.135	1.335	1.503	1.059	1.206	1.130	1.310	1.204	0.979	1.226	1.046
	[0.314]	[0.213]	[0.411]	[0.264]	[0.44]	[0.559]	[0.543]	[0.443]	[0.46]	[0.314]	[0.411]	[0.327]	[0.473]	[0.381]	[0.481]	[0.394]	[0.238]	[0.137]	[0.135]	[0.081]
1996	1.442	1.385	1.455	1.318	0.936	1.700	1.211	1.440	1.175	1.135	1.319	1.530	1.053	1.129	1.134	1.215	1.210	0.993	1.226	1.046
	[0.314]	[0.213]	[0.411]	[0.263]	[0.441]	[0.566]	[0.541]	[0.455]	[0.46]	[0.314]	[0.406]	[0.333]	[0.496]	[0.376]	[0.507]	[0.384]	[0.242]	[0.14]	[0.135]	[0.081]
1997	1.449	1.390	1.469	1.590	0.937	1.730	1.122	1.515	1.168	1.219	1.325	1.545	1.051	1.131	1.106	1.106	1.210	0.993	1.226	1.046
	[0.316]	[0.214]	[0.424]	[0.324]	[0.441]	[0.576]	[0.56]	[0.535]	[0.441]	[0.325]	[0.321]	[0.264]	[0.495]	[0.376]	[0.521]	[0.368]	[0.242]	[0.14]	[0.135]	[0.081]
1998	1.449	1.390	1.475	1.598	0.940	1.755	1.122	1.515	1.168	1.219	1.328	1.554	1.051	1.130	1.106	1.120	1.210	0.993	1.226	1.046
	[0.316]	[0.214]	[0.425]	[0.326]	[0.443]	[0.584]	[0.56]	[0.535]	[0.441]	[0.325]	[0.322]	[0.266]	[0.495]	[0.376]	[0.521]	[0.373]	[0.242]	[0.14]	[0.135]	[0.081]
1999	1.452	1.391	1.475	1.598	0.943	1.768	1.122	1.515	1.193	1.330	1.323	1.564	1.051	1.130	1.109	1.113	1.219	0.989	1.230	1.045
	[0.316]	[0.214]	[0.425]	[0.326]	[0.444]	[0.589]	[0.56]	[0.535]	[0.435]	[0.343]	[0.325]	[0.272]	[0.495]	[0.376]	[0.522]	[0.37]	[0.248]	[0.142]	[0.136]	[0.082]
2000	1.453	1.388	1.475	1.598	0.946	1.780	0.760	1.275	1.196	1.303	1.588	1.774	1.105	0.909	1.125	1.124	1.217	0.978	1.230	1.045
	[0.317]	[0.214]	[0.425]	[0.326]	[0.445]	[0.593]	[0.537]	[0.637]	[0.436]	[0.336]	[0.39]	[0.308]	[0.59]	[0.343]	[0.53]	[0.374]	[0.248]	[0.141]	[0.136]	[0.082]
2001	1.453	1.388	1.476	1.586	0.969	1.693	0.770	1.235	1.196	1.303	1.586	1.773	1.111	0.923	1.123	1.185	1.242	0.970	1.230	1.045
	[0.317]	[0.214]	[0.426]	[0.323]	[0.457]	[0.564]	[0.544]	[0.617]	[0.436]	[0.336]	[0.39]	[0.308]	[0.593]	[0.348]	[0.529]	[0.394]	[0.261]	[0.144]	[0.136]	[0.082]
2002	1.444	1.318	1.405	1.461	0.972	1.685	1.155	1.234	1.196	1.303	1.306	1.380	1.113	0.921	1.124	1.193	1.242	0.970	1.224	1.043

	[0.322]	[0.208]	[0.397]	[0.292]	[0.457]	[0.561]	[0.577]	[0.436]	[0.436]	[0.336]	[0.435]	[0.325]	[0.594]	[0.347]	[0.529]	[0.397]	[0.261]	[0.144]	[0.136]	[0.082]
2003	1.433	1.323	1.454	1.509	0.973	1.668	1.152	1.333	1.184	1.213	1.430	1.603	1.060	1.220	1.124	1.200	1.242	0.970	1.196	1.000
	[0.312]	[0.204]	[0.411]	[0.301]	[0.458]	[0.555]	[0.615]	[0.503]	[0.447]	[0.324]	[0.396]	[0.314]	[0.474]	[0.385]	[0.529]	[0.399]	[0.261]	[0.144]	[0.13]	[0.077]
2004	1.456	1.388	1.454	1.509	0.974	1.660	1.152	1.333	1.175	1.203	1.430	1.603	1.059	1.221	1.121	1.213	1.242	0.970	1.233	1.081
	[0.329]	[0.222]	[0.411]	[0.301]	[0.459]	[0.553]	[0.615]	[0.503]	[0.444]	[0.321]	[0.396]	[0.314]	[0.473]	[0.386]	[0.528]	[0.404]	[0.261]	[0.144]	[0.132]	[0.081]
2005	1.456	1.388	1.448	1.510	0.974	1.663	1.131	1.290	1.175	1.203	1.274	1.338	1.057	1.225	1.119	1.221	1.244	0.965	1.226	1.076
	[0.329]	[0.222]	[0.409]	[0.301]	[0.459]	[0.554]	[0.604]	[0.487]	[0.444]	[0.321]	[0.437]	[0.324]	[0.472]	[0.387]	[0.527]	[0.406]	[0.265]	[0.145]	[0.134]	[0.083]
2006	1.445	1.398	1.445	1.509	0.974	1.664	1.125	1.276	1.168	1.198	1.254	1.321	1.057	1.224	1.118	1.236	1.244	0.965	1.224	1.098
	[0.327]	[0.223]	[0.408]	[0.301]	[0.459]	[0.554]	[0.601]	[0.482]	[0.441]	[0.32]	[0.43]	[0.32]	[0.472]	[0.387]	[0.527]	[0.412]	[0.265]	[0.145]	[0.13]	[0.082]
2007	1.445	1.398	1.442	1.583	0.974	1.663	1.121	1.269	1.151	1.220	1.258	1.391	1.099	0.910	1.114	1.249	1.241	0.976	1.229	1.078
	[0.327]	[0.223]	[0.416]	[0.323]	[0.459]	[0.554]	[0.598]	[0.479]	[0.435]	[0.326]	[0.419]	[0.327]	[0.587]	[0.343]	[0.525]	[0.416]	[0.264]	[0.147]	[0.132]	[0.082]

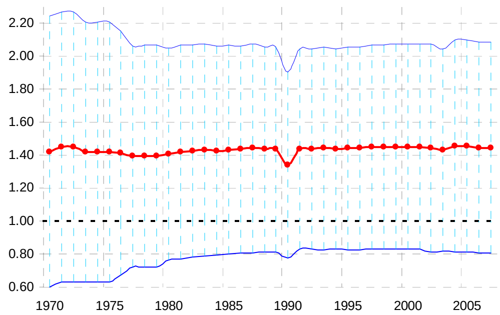
Table A.5.2.1. *The dynamics of the regression estimate of the Pareto exponent*



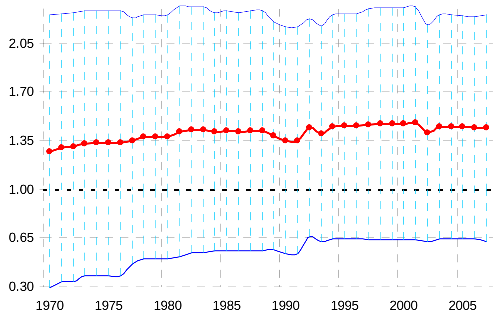
a. Russian Federation



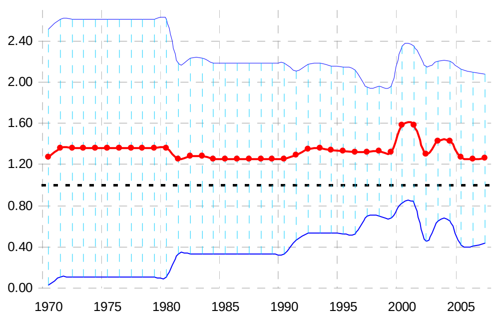
b. Ukraine



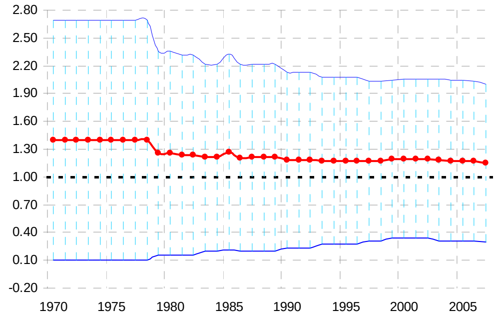
c. Poland



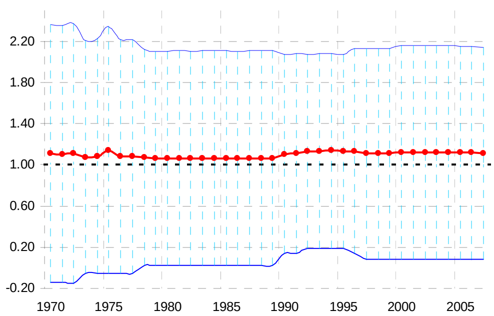
d. Romania



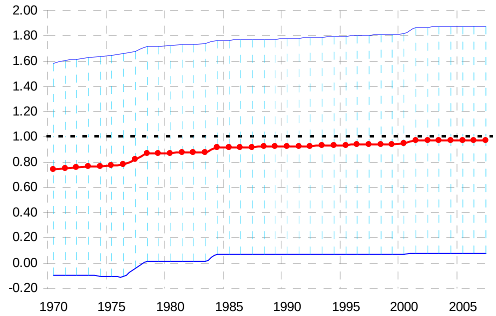
e. Former Yugoslavia



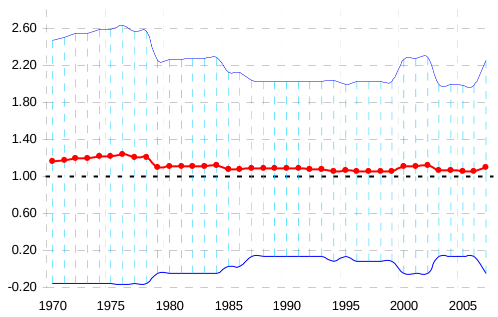
f. Belarus



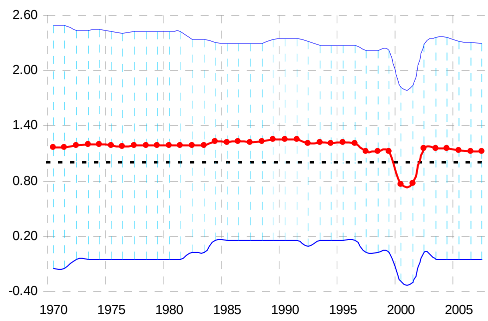
g. Baltic States



h. Hungary

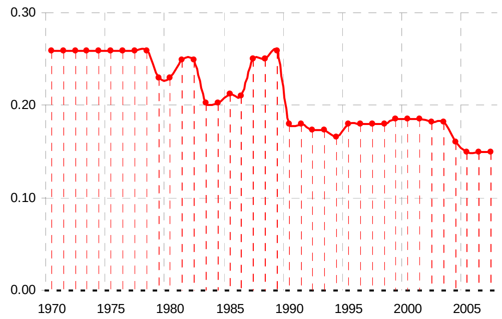


i. Former Czechoslovakia

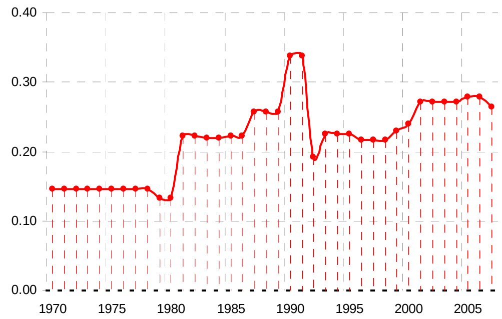


j. Bulgaria

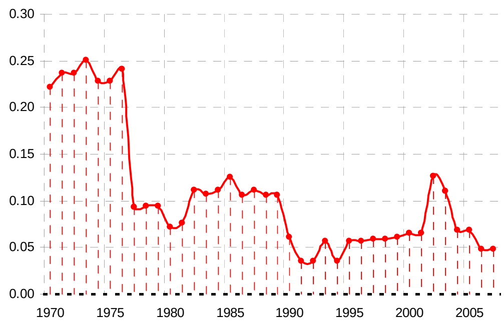
Table A.5.2.1. *The dynamics of the difference between the regression and the MLE estimates of the Pareto exponent*



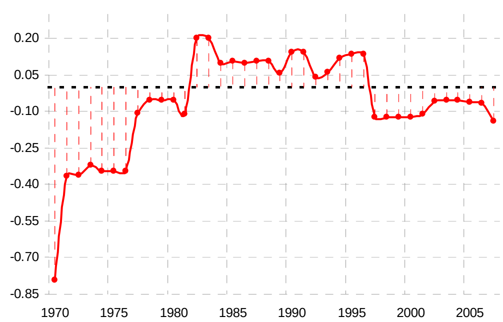
a. Russian Federation



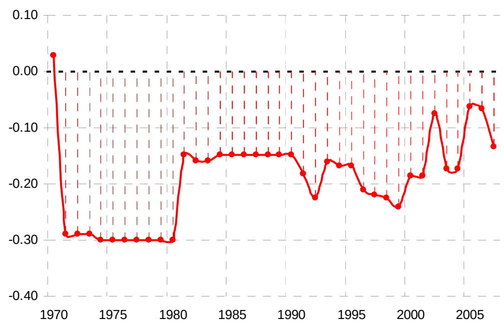
b. Ukraine



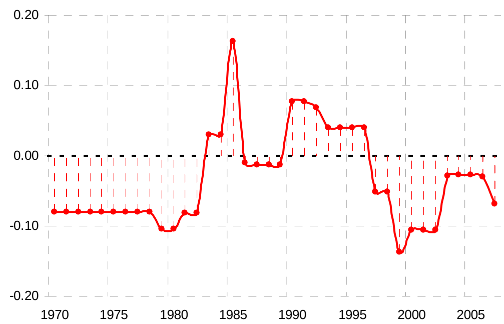
c. Poland



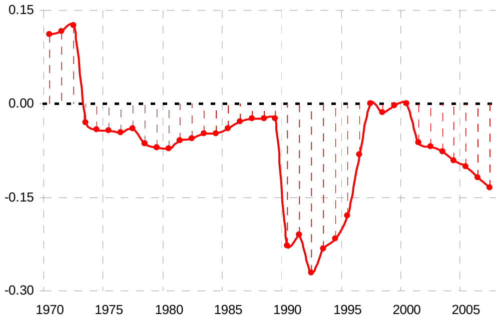
d. Romania



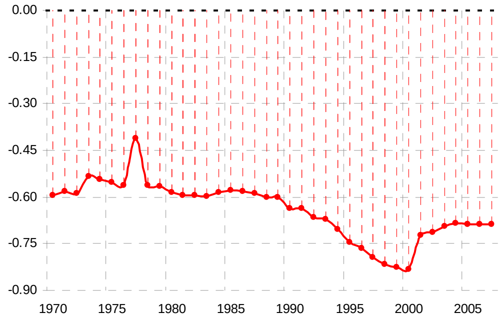
e. Former Yugoslavia



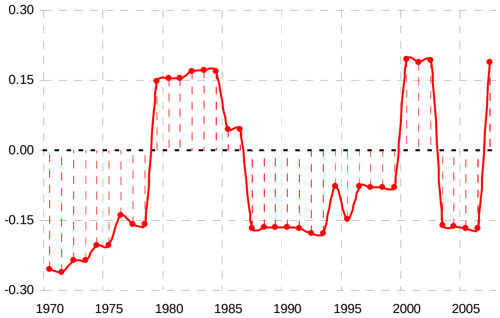
f. Belarus



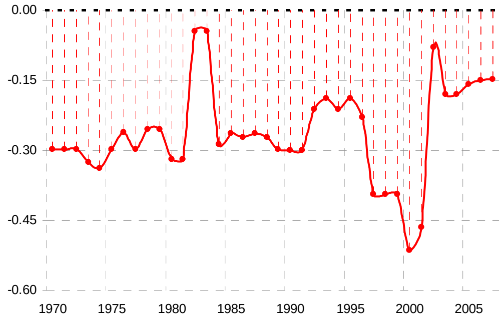
g. Baltic States



h. Hungary



i. Former Czechoslovakia



j. Bulgaria

Table A5.3.1. Parameters of regression of logarithms of city ranks *i* for largest cities in Russia (with the population above 100 thousand people) against the logarithms of city size *Ni*:

$$\ln(i-1/2) = a - \zeta \cdot \ln Ni$$

Dependent variable	Logarithm of city ranks ln(*i*-1/2)				
Years	**1897**	**1926**	**1939**	**1959**	**1970**
Independent variable	Regression coefficient				
Constant	5.589910 (0.748386)	7.920504 (0.487601)	9.900424 (0.258400)	10.59869 (0.185453)	10.96055 (0.209420)
$\ln N_i$	-0.824135 (0.135973)	-1.113586 (0.090925)	-1.275446 (0.046920)	-1.303802 (0.032391)	-1.297354 (0.035298)
R^2	0.860	0.893	0.938	0.961	0.949
$F(R^2)$	36.74	150.00	738.94	1620.21	1350.90
Sample size	*n*=8	*n*=20	*n*=51	*n*=66	*n*=75
Years	**1979**	**1989**	**2002**	**2003**	**2004**
Independent variable	Regression coefficient				
Constant	11.01538 (0.116334)	11.00539 (0.124625)	10.92802 (0.113270)	10.92045 (0.113024)	10.93742 (0.112480)
$\ln N_i$	-1.266171 (0.020597)	-1.237672 (0.021873)	-1.227856 (0.020085)	-1.226676 (0.020043)	-1.229840 (0.019950)
R^2	0.965	0.956	0.960	0.960	0.960
$F(R^2)$	3778.86	3201.90	3737.37	3745.54	3800.33
Sample size	*n*=138	*n*=151	*n*=159	*n*=159	*n*=159
Years	**2005**	**2006**	**2007**	**2008**	**2009**
Independent variable	Regression coefficient				
Constant	10.96038 (0.108596)	10.97003 (0.109227)	10.96880 (0.108756)	10.96678 (0.108854)	10.95741 (0.107797)
$\ln N_i$	-1.233169 (0.019318)	-1.234586 (0.019426)	-1.234937 (0.019351)	-1.234633 (0.019369)	-1.232836 (0.019195)
R^2	0.962	0.962	0.962	0.962	0.962
$F(R^2)$	4075.05	4039.18	4072.86	4063.22	4125.06
Sample size	*n*=163	*n*=163	*n*=163	*n*=163	*n*=164

Table A5.3.2. Parameters of the regression of the logarithm of the population of Russian cities (except for Moscow and Saint-Petersburg) against their rank for the years 1897-2009

Dependent Variable	Logarithm of the population N_i					
Independent variable	Regression coefficient					
	1897	**1926**	**1939**	**1959**	**1970**	**1979**
Const	5.078591	5.819091	6.462330	6.801082	6.959886	6.715370
i	-0.040165	-0.049725	-0.041254	-0.036110	-0.030297	-0.016405
R^2...	0.848	0.888	0.857	0.854	0.900	0.943
$F(R^2)$	518.01	562.24	449.02	439.91	677.90	2610.93
Included	95	73	77	77	77	159

observations						

Dependent Variable	Logarithm of the population N_i					
Independent variable	Regression coefficient					
	1989	**2002**	**2003**	**2004**	**2005**	**2006**
Const	6.823465	6.761217	6.757072	6.755904	6.736914	6.736782
i	-0.015847	-0.015022	-0.014964	-0.014973	-0.014572	-0.014553
R^2...	0.948	0.956	0.956	0.956	0.950	0.951
$F(R^2)$	2894.26	3478.67	3488.50	3423.99	3062.22	3082.90
Included observations	162	161	162	161	162	162

Dependent Variable	Logarithm of the population N_i		
Independent variable	Regression coefficient		
	2007	**2008**	**2009**
Const	6.733465	6.733453	6.734632
i	-0.014545	-0.014547	-0.014545
R^2...	0.950	0.951	0.950
$F(R^2)$	3059.02	3073.39	3046.31
Included observations	162	162	162

Note: The coefficients are significant if the significance level is above 0.00005. R^2 is significant if the significance level is not larger than 0.0000005.

Table A5.3.3. *Parameters of regression of* ***c*** *and* ***k*** *agaist ranks Years and political variables* ***P*1**, ***P*2, *P*3** for the cities in Russia

Dependent variable	*c*	*c*	*c*	*c*	*k*
	Model 1	**Model 2**	**Model 3**	**Model 4**	
Independent variable	Regression coefficient				Regression coefficient
Const	-4.889177 (11.36847)	5.078591 (0.146145)	-31.47273 (4.668597)	---	-1.349853 (0.171911)
t	0.005254 (0.005992)	---	0.019399 (0.002392)	0.002678 (7.55E-05)	0.000690 (9.06E-05)
*P*1	0.875585 (0.280179)	1.062120 (0.178990)	---	0.966226 (0.177637)	-0.029829 (0.004237)
*P*2	0.464865 (0.281449)	0.630060 (0.111620)	---	0.572450 (0.124103)	-0.008002 (0.004256)
*P*3	-0.245474 (0.208275)	---	-0.687989 (0.163097)	-0.164946 (0.087757)	-0.011622 (0.003149)
R^2	0.933	0.923	0.869	0.932	0.977
$F(R^2)$	35.09	72.02	39.64	---	106.57
DW	2.613	2.743	1.179	2.859	2.329
Sample size	*n*=15				

Note: Standard errors of the regression coefficients are given in brackets.

Table A5.3.4. Parameters of the regression of the logarithm of the population of Belarusian cities against their rank (ln*Size*=*C*+*κ Rank*) for the years 1970-2009

Years	***N***	**Variable**	**Coefficient**	**Std.Error**	***t*-statistic**	***p***	R^2	***F*-statistic**
1970	198	*Rank*	-0.018014	0.0006	-29.560	0.0000	0.817	873.80
		C	3.656932	0.0699	52.297	0.0000		
1979	200	*Rank*	-0.019605	0.0006	-33.610	0.0000	0.851	1129.62
		C	4.017004	0.0676	59.418	0.0000		
1989	202	*Rank*	-0.020990	0.0006	-37.683	0.0000	0.877	1420.02
		C	4.333892	0.0652	66.468	0.0000		
1990	202	*Rank*	-0.021041	0.0006	-37.946	0.0000	0.878	1439.91
		C	4.366013	0.0649	67.264	0.0000		
1991	202	*Rank*	-0.021133	0.0006	-38.365	0.0000	0.880	1471.90
		C	4.382248	0.0645	67.963	0.0000		
1992	202	*Rank*	-0.021238	0.0006	-38.535	0.0000	0.881	1484.93
		C	4.398832	0.0645	68.183	0.0000		
1993	202	*Rank*	-0.021356	0.0005	-39.016	0.0000	0.884	1522.22
		C	4.416907	0.0641	68.936	0.0000		
1994	202	*Rank*	-0.021437	0.0005	-39.377	0.0000	0.886	1550.55
		C	4.432715	0.0637	69.557	0.0000		
1995	202	*Rank*	-0.021431	0.0005	-39.341	0.0000	0.886	1547.74
		C	4.437681	0.0638	69.593	0.0000		
1997	203	*Rank*	-0.021565	0.0005	-40.505	0.0000	0.891	1640.68
		C	4.457189	0.0626	71.168	0.0000		
1998	20	*Rank*	-0.021255	0.0005	-39.865	0.000	0.88	1589.2

	5					0	7	2
		C	4.434770	0.0633	70.021	0.0000		
1999	205	*Rank*	-0.021485	0.0005	-40.402	0.0000	0.889	1632.32
		C	4.423677	0.0632	70.029	0.0000		
2000	205	*Rank*	-0.021539	0.0005	-40.513	0.0000	0.890	1641.34
		C	4.428232	0.0632	70.118	0.0000		
2001	207	*Rank*	-0.021282	0.0005	-40.434	0.0000	0.889	1634.87
		C	4.414205	0.0631	69.921	0.0000		
2002	207	*Rank*	-0.021354	0.0005	-40.476	0.0000	0.889	1638.30
		C	4.416414	0.0633	69.792	0.0000		
2003	206	*Rank*	-0.021361	0.0005	-40.089	0.0000	0.887	1607.16
		C	4.412470	0.0636	69.376	0.0000		
2004	206	*Rank*	-0.021384	0.0005	-40.065	0.0000	0.887	1605.22
		C	4.409145	0.0637	69.207	0.0000		
2005	206	*Rank*	-0.021489	0.0005	-40.056	0.0000	0.887	1604.48
		C	4.410506	0.0640	68.874	0.0000		
2006	206	*Rank*	-0.021573	0.0005	-40.143	0.0000	0.888	1611.42
		C	4.411310	0.0641	68.767	0.0000		
2007	207	*Rank*	-0.021660	0.0005	-40.484	0.0000	0.889	1638.99
		C	4.414477	0.0642	68.791	0.0000		
2008	206	*Rank*	-0.021725	0.0005	-40.246	0.0000	0.888	1619.75
		C	4.416968	0.0644	68.549	0.0000		
2009	206	*Rank*	-0.021776	0.0005	-40.365	0.0000	0.889	1629.33
		C	4.421704	0.0644	68.665	0.0000		

Table A5.3.5. Parameters of the regression of the logarithm of the population of Belarusian cities against their rank (ln*Size*=*C*+*κ Rank*) for the years 1970-2009 without MINSK

Years	***N***	**Variable**	**Coefficient**	**Std.Error**	***t*-statistic**	***p***	**R^2**	***F*-statistic**
1970	197	*Rank*	-0.017521	0.0005	-32.184	0.0000	0.842	1035.83
		C	3.591554	0.0626	57.349	0.0000		
1979	199	*Rank*	-0.019126	0.0005	-36.860	0.0000	0.873	1358.63
		C	3.952871	0.0603	65.563	0.0000		
1989	201	*Rank*	-0.020532	0.0005	-41.469	0.0000	0.896	1719.66
		C	4.271895	0.0581	73.525	0.0000		
1990	201	*Rank*	-0.020587	0.0005	-41.711	0.0000	0.897	1739.80
		C	4.304593	0.0579	74.320	0.0000		
1991	201	*Rank*	-0.020689	0.0005	-42.012	0.0000	0.899	1765.03
		C	4.322171	0.0578	74.792	0.0000		
1992	201	*Rank*	-0.020786	0.0005	-42.394	0.0000	0.900	1797.24
		C	4.337667	0.0575	75.389	0.0000		
1993	201	*Rank*	-0.020905	0.0005	-42.982	0.0000	0.903	1847.45
		C	4.355911	0.0571	76.320	0.0000		
1994	201	*Rank*	-0.020989	0.0005	-43.402	0.0000	0.904	1883.74
		C	4.371978	0.0567	77.042	0.0000		
1995	201	*Rank*	-0.020982	0.0005	-43.351	0.0000	0.904	1879.29
		C	4.376965	0.0568	77.062	0.0000		
1997	202	*Rank*	-0.021123	0.0005	-44.775	0.0000	0.909	2004.82
		C	4.397009	0.0556	79.039	0.0000		
1998	204	*Rank*	-0.020815	0.0005	-43.967	0.0000	0.905	1933.12
		C	4.374362	0.0564	77.594	0.000		

						0		
1999	204	*Rank*	-0.021047	0.0005	-44.558	0.0000	0.908	1985.44
		C	4.363493	0.0562	77.578	0.0000		
2000	204	*Rank*	-0.021101	0.0005	-44.687	0.0000	0.908	1996.91
		C	4.368049	0.0562	77.683	0.0000		
2001	206	*Rank*	-0.020849	0.0005	-44.567	0.0000	0.907	1986.19
		C	4.354215	0.0562	77.411	0.0000		
2002	206	*Rank*	-0.020921	0.0005	-44.609	0.0000	0.907	1989.95
		C	4.356310	0.0564	77.256	0.0000		
2003	205	*Rank*	-0.020921	0.0005	-44.217	0.0000	0.906	1955.13
		C	4.351832	0.0566	76.865	0.0000		
2004	205	*Rank*	-0.020943	0.0005	-44.217	0.0000	0.906	1955.11
		C	4.348269	0.0567	76.723	0.0000		
2005	205	*Rank*	-0.021046	0.0005	-44.194	0.0000	0.906	1953.10
		C	4.349380	0.0570	76.327	0.0000		
2006	205	*Rank*	-0.021129	0.0005	-44.300	0.0000	0.906	1962.53
		C	4.350032	0.0571	76.222	0.0000		
2007	206	*Rank*	-0.021219	0.0005	-44.648	0.0000	0.907	1993.46
		C	4.353375	0.0571	76.184	0.0000		
2008	205	*Rank*	-0.021279	0.0005	-44.416	0.0000	0.907	1972.76
		C	4.355422	0.0573	75.975	0.0000		
2009	205	*Rank*	-0.021329	0.0005	-44.570	0.0000	0.907	1986.45
		C	4.360096	0.0573	76.140	0.0000		

Table A5.3.6. *Estimation results for the regression ln $N_i = c+k{\cdot}i$ of the population of Central Asian cities in 1999*

Dependent variable	ln N_i
Independent variable	Regression coefficient
Constant	13.36066 (0.081707)
i	-0.045002 (0.003093)
R^2	0.831
$F(R^2)$	211.64
Sample size	*n*=45

Note: Standard errors of the regression coefficients are given in brackets. All the coefficients are significant at the significance level of 0.00005.

Table A5.3.7. *Estimates for the regression ln $N_i = c+k{\cdot}i$ for cities of Central Asia in 1970-2006*

Dependent variable	log of the population N_i				
	1970	1971	1975	1980	1985
Independent variable	Regression coefficient				
Constant	13.21387 (0.1064)	13.24355 (0.1057)	13.30165 (0.0973)	13.32473 (0.0858)	13.39433 (0.0852)
Rank	-0.068835 (0.0064)	-0.068835 (0.0064)	-0.063167 (0.0053)	-0.053994 (0.0038)	-0.051997 (0.0037)
R^2	0.816	0.818	0.830	0.846	0.841
$F(R^2)$	115.30	116.82	141.69	198.00	195.94
Sample size	28	28	31	38	39

Independent variable	1987	1990	1999	2006
	Regression coefficient			
Constant	13.40747 (0.0830)	13.41749 (0.0855)	13.36066 (0.0817)	13.48998 (0.0842)
Rank	-0.049425 (0.0034)	-0.050689 (0.003633)	-0.045002 (0.003093)	-0.051685 (0.0035)
R^2	0.841	0.837	0.831	0.849
$F(R^2)$	205.95	194.69	211.64	219.04
Sample size	41	40	45	41

Note: Standard errors of the regression coefficients are given in brackets. All the coefficients are significant at the significance level of 0.00005.

Table A5.3.8. *Parameters of the regression of* ***c*** *and* ***k*** *on the time trend* ***t*** *and the political variable P for cities of Central Asia in 1970-2006*

Dependent variable	***c***	***k***
Independent variable	Regression coefficient	
Const	-7.601766 (2.123907)	-1.877394 (0.443553)
t	0.010573 (0.001073)	0.000919 (0.000224)
P	-0.144598 (0.029940)	-0.011148 (0.006253)
R^2	0.955	0.801
$F(R^2)$	64.08	12.09
DW	1.947	2.422
Sample size	*n*=9	

Note: Standard errors of the regression coefficients are given in brackets.

Table A5.3.9. *Estimation results for the regression ln N_i = c+k·i of the population of Caucasus cities in 2007*

Dependent variable	ln *Ni*
Independent variable	Regression coefficient
Constant	14.50335 (0.240176)
i	-0.336194 (0.038708)
R^2	0.904
$F(R^2)$	75.44
Sample size	*n*=10

Note: Standard errors of the regression coefficients are given in brackets. All the coefficients are significant at the significance level of 0.00005.

Table A5.3.10. *Parameters of regression of logarithms of the population N_i for cities of Caucasus agaist its ranks: ln N_i =c+k·i*

Dependent variable	log of the population N_i			
	1970	1971	1975	1980
Independent variable	Regression coefficient			
Constant	13.86252 (0.2725)	13.88072 (0.2747)	13.98215 (0.2970)	14.12501 (0.2799)
Rank	-0.276535 (0.0439)	-0.274877 (0.0443)	-0.257304 (0.0438)	-0.261800 (0.0413)
R^2	0.832	0.828	0.793	0.817
$F(R^2)$	39.63	38.56	34.52	40.24
Sample size	10	10	11	11
Independent variable	1985	1987	1990	2007
	Regression coefficient			
Constant	14.21326 (0.2773)	14.24711 (0.2751)	14.29858 (0.2583)	14.50335 (0.2402)
Rank	-0.262220 (0.0409)	-0.262647 (0.0406)	-0.299119 (0.0416)	-0.336194 (0.0387)
^{2}	0.820	0.823	0.866	0.904
$F(R^2)$	41.14	41.93	51.62	75.44
Sample size	11	11	10	10

Table A5.3.11. *Parameters of the regression of* ***c*** *and* ***k*** *on the time trend* ***t*** *and the political variable P for cities of Caucasus in 1970-2007*

Dependent variable	***c***	***k***
Independent variable	Regression coefficient	
Const	-30.22902 (1.815403)	-0.270643 (0.005465)
Year	0.022385 (0.000917)	---
P	-0.194493 (0.031475)	-0.065551 (0.015458)
R^2	0.995	0.750
$F(R^2)$	535.60	17.98
DW	1.4994	1.9714
Sample size	*n*=8	

Note: Standard errors of the regression coefficients are given in brackets.

Table A5.3.12. Parameters of the regression of logarithms $\ln^4$ of the population Ni for the populated areas of Russia (except for Moscow and Saint-Petersburg) in the years 1897-2009 against their ranks i: $\ln^4 Ni = c+k\cdot i$

Dependent Variable	$\ln^4 N_i$					
Independent variable	Regression coefficient					
	1897	**1926**	**1939**	**1959**	**1970**	**1979**
Const	-0.162999	-0.432231	-0.310213	-0.289375	-0.351581	-0.396210
i	-0.043392	-0.019682	-0.013837	-0.010744	-0.006421	-0.003505
R^2...	0.760	0.812	0.631	0.323	0.655	0.975
$F(R^2)$	206.36	250.31	123.29	35.36	142.61	6090.79
Sample size	67	60	74	76	77	158
Dependent Variable	$\ln^4 N_i$					
Independent variable	Regression coefficient					
	1989	**2002**	**2003**	**2004**	**2005**	**2006**
Const	-0.365500	-0.400339	-0.400607	-0.401202	-0.407275	-0.407282
i	-0.003580	-0.003003	-0.003001	-0.002994	-0.002870	-0.002866
R^2...	0.556	0.986	0.986	0.985	0.994	0.994
$F(R^2)$	200.50	11079.18	11666.09	10506.97	26019.70	25547.44
Sample size	162	161	162	161	162	162
Dependent Variable	$\ln^4 N_i$					
Independent variable	Regression coefficient					
	2007	**2008**	**2009**			
Const	-0.407732	-0.407672	-0.407659			
i	-0.002867	-0.002868	-0.002865			
R^2...	0.994	0.994	0.994			
$F(R^2)$	25553.24	26221.30	26060.48			
Sample size	162	162	162			

Table A5.3.12. Parameters of the regression coefficient c_4 of the equation $\ln^4 N_i = c_4 + k_4 i$ for time t (except for Moscow and Saint-Petersburg)

Dependent variable	C_4
Independent variable	Regression coefficient
Const	-0.258998 (0.032879)
t	-0.001264 (0.000285)
R^2	0.767
$F(R^2)$	19.72
DW	1.479
Sample size	n=8 (2002-2009 years)

Note. Standard errors of the regression coefficients are given in brackets. The regression coefficients are significant at the significance level not larger than 0.0045; R^2 is significant at the significance level not larger than 0.0044.

Table A5.3.13. Parameters of the regression coefficients k_4 of the equation $\ln^4 N_i = c_4 + k_4 i$ for time t (except for Moscow and Saint-Petersburg)

Dependent variable	k_4
Independent variable	Regression coefficient
Const	-0.071110 (0.001650)
ln$t$	0.014469 (0.000375)
R^2	0.991347
$F(R^2)$	1489.387
DW	1.114548
Sample size	n=15

Note. Standard errors of the regression coefficients are given in brackets. The regression coefficients are significant at the significance level not larger than 0.0000005; R^2 is significant at the significance level not larger than 0.0000005

Table A5.3.14. *Regression $\ln^r N_i = c+k\cdot i$ of the logarithm iterations $\ln^r N_i$ on the ranks i of city sizes N_i of the Central Asian* cities *in 1999*

Dependent variable	Hierarchy of logarithms of the population N_i			
	Ln(N_i)	$\text{Ln}^2(N_i)$	$\text{Ln}^3(N_i)$	$\text{Ln}^4(N_i)$
Independent variable	Regression coefficient	Regression coefficient	Regression coefficient	Regression coefficient
Constant	13.36066 (0.081707)	2.592944 (0.005919)	0.952914 (0.002253)	-0.048076 (0.002331)
Rank	-0.045002 (0.003093)	-0.003590 (0.000224)	-0.001421 (8.53E-05)	-0.001534 (8.83E-05)
R^2	0.831	0.856	0.866	0.875
$F(R^2)$	211.64	256.62	277.41	301.98
Sample size	n=45			

Note: Standard errors of the regression coefficients are given in brackets. All the coefficients are significant at the significance level of 0.00005.

Table A5.3.15. *Regression $\ln^r N_i = c+k\cdot i$ of the logarithm iterations $\ln^r N_i$ on the ranks i of city sizes N_i of the Caucasus in 2007*

Dependent variable	Hierarchy of logarithms of the population N_i			
	Ln(N_i)	$\text{Ln}^2(N_i)$	$\text{Ln}^3(N_i)$	$\text{Ln}^4(N_i)$
Independent variable	Regression coefficient			
Constant	14.50335 (0.240176)	2.678939 (0.017464)	0.986183 (0.006662)	-0.013023 (0.006911)
Rank	-0.336194 (0.038708)	-0.026196 (0.002815)	-0.010276 (0.001074)	-0.010991 (0.001114)
R^2	0.904	0.915	0.920	0.924
$F(R^2)$	75.44	86.62	91.62	97.39
Sample size	n=10			

Note: Standard errors of the regression coefficients are given in brackets. All the coefficients are significant at the significance level of 0.00005.

Table A.5.4.1. Description of the dataset for the "within" distribution analysis

Country	Investigated period by decades	Numbers of cities
Poland	1961-2004	890
Belarus	1970-2009	207
Hungary	1970-2001	237
Russia	1897-2002	479

Table A.5.4.2. Values of LR statistics to test Markovity of Polish cities distribution

Years	1961	1974	1985	1994
LR(O(0))	1943.578	1966.536	2562.915	2880.135
LR(O(1))	-396.545	-402.227	-478.677	

Table A.5.4.3. The probability of acceptance of Markovity of appropriate order in Poland

Years	DF	1961	1974	1985	1994
0 order Markovity	36	0	0	0	0
≥ 1 order Markovity	28	1	1	1	1

DF - Degrees of freedom

Table A.5.4.4. Values of LR statistics to test Markovity of Belarusian cities distribution

Years	1970	1979	1989	1999
LR(O(0))	478.2052689	563.5174	566.3945	548.5899
LR(O(1))	-100.6281285	-103.289	-115.623	

Table A.5.4.5. The probability of acceptance of Markovity of appropriate order in Belarus

Years	DF	1970	1979	1989	1999
0 order Marcovity	36	1.19068E-78	5.72E-96	1.48E-96	6.33E-93
≥ 1 order Marcovity	14	1	1	1	1

DF - Degrees of freedom

Table A.5.4.6. Values of LR statistics to test Markovity of Hungarian cities distribution

Years	1880	1890	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2001
LR(O(0))	662	693	558	662	698	672	642	657	501	555	570	689	702
LR(O(1))	6.55	12.8	8.3	5.4	5.7	10.3	16.4	23.5	13.8	5.9	-0.64721	1.73	

Table A.5.4.7. The probability of acceptance of Markovity of appropriate order in Hungary (1880-1940)

Years	DF	1880	1890	1900	1910	1920	1930	1940
0 order Marcovity	36	2.9E-116	1.5E-122	6.3E-95	2.9E-116	1.2E-123	2.5E-118	3.6E-112
≥ 1 order Marcovity	22	0.9993	0.9395	0.9963	0.9998	0.9998	0.983	0.7955

DF - Degrees of freedom

Table A.5.4.8. The probability of acceptance of Markovity of appropriate order in Hungary (1950-2001)

Years	DF	1950	1960	1970	1980	1990	2001
0 order Marcovity is	36	4E-115	2.74E-83	2.93E-94	2.22E-97	1E-121	2.2E-124
≥ 1 order Marcovity	22	0.374	0.90	0.9997	1	1	

DF - Degrees of freedom

Table A.5.4.9. Values of LR statistics to test Markovity of Russian cities distribution

Years	1897	1926	1939	1959	1970	1979
LR(O(0))	605.109	599.7309	938.1978	1211.197	1340.7	1358.753
LR(O(1))	45.33589	64.50166	17.65158	36.06105	43.01439	41.3714

Table A.5.4.10. The probability of acceptance of Markovity of appropriate order in Russia

Years	DF	1897	1926	1939	1959	1970	1979
0 order Marcovity	36	1.8E-104	2.2E-103	1.4E-173	5.6E-231	2.4E-258	3.6E-262
≥ 1 order Marcovity	67	0.980458	0.563856	1	0.9992	0.99007	0.99418

Table A.5.4.11. Probability transition matrix for Poland, 1961-2004

	1	2	3	4	5	6	7	Number of observations
	<10%	<20%	<30%	<50%	<100%	<200%	>200%	
1	0.786	0.155	0.024	0.0065	0.026	0.002	0	459
2	0.072	0.838	0.082	0.004	0.0028	0.0014	0	722
3	0.004	0.123	0.73	0.14	0.002	0	0	480
4	0.002	0	0.0687	0.77	0.147	0.0076	0	524
5	0.003	0.003	0	0.027	0.888	0.072	0.005	582
6	0.004	0.0035	0	0.0035	0.042	0.866	0.081	284
7	0	0	0	0	0	0.02	0.979	290

Table A.5.4.12. Probability transition matrix for Belarus, 1970-2009

	1	2	3	4	5	6	7	Number of observations
	<10%	<20%	<30%	<50%	<100%	<200%	>200%	
1	0.944	0.043	0.012	0	0	0	0	162
2	0.265	0.649	0.086	0	0	0	0	151
3	0	0.106	0.807	0.087	0	0	0	161
4	0	0	0.128	0.832	0.040	0	0	149
5	0	0	0	0.149	0.824	0.027	0	74
6	0	0	0	0	0.098	0.854	0.049	41
7	0	0	0	0	0	0.027	0.973	75

Table A.5.4.13. Probability transition matrix for Hungary, 1970-2001

	1	2	3	4	5	6	7	Number of observations
	<10%	<20%	<30%	<50%	<100%	<200%	>200%	
1	0.87	0.12	0	0.01	0	0	0	151
2	0.03	0.88	0.077	0.003	0.005	0.003	0	376
3	0	0.086	0.82	0.09	0.003	0.001	0	427
4	0	0	0.1	0.85	0.05	0	0	729
5	0	0	0	0.1	0.88	0.02	0	786
6	0	0	0	0	0.09	0.88	0.03	388
7	0	0	0	0	0	0.08	0.92	224

Table A.5.14. Probability transition matrix for Russia, 1897-2002

	1	2	3	4	5	6	7	Number of observations
	<10%	<20%	<30%	<50%	<100%	<200%	>200%	
1	0.92	0.05	0.017	0.011	0.002	0	0	524
2	0.179	0.736	0.057	0.021	0.006	0	0.001	700
3	0.022	0.330	0.525	0.100	0.022	0	0	448
4	0.002	0.057	0.232	0.609	0.092	0.007	0.002	557
5	0	0.016	0.028	0.220	0.654	0.069	0.014	509
6	0	0	0	0.004	0.152	0.726	0.119	270
7	0	0	0	0	0.003	0.061	0.936	345

Table A.5.4.2.15. Mean first passage time matrix for Poland, years

Class	1	2	3	4	5	6	7
	<10%	<20%	<30%	<50%	<100%	<200%	>200%
1	588	920	115	850	417	536	827
2	2053	260	739	689	430	550	843
3	3438	1890	476	380	340	470	760
4	4659	3480	2600	340	188	340	610
5	5173	413	3690	1955	100	226	487
6	5556	4530	4160	2590	1020	60	290
7	6060	5020	4630	3076	1520	470	17

Table A.5.4.2.16. Mean first passage time matrix for Belarus, years

Class	1	2	3	4	5	6	7
	<10%	<20%	<30%	<50%	<100%	<200%	>200%
1	18	220	490	1085	3200	15840	40077
2	99	80	400	994	3110	15750	39980
3	290	190	63	597	2716	15340	39585
4	410	300	117	91	212	14720	38980
5	530	420	238	120	330	12510	36830
6	820	700	529	410	290	1250	24620
7	1190	1070	907	780	660	386	670

Table A.5.4.2.17. Mean first passage time matrix for Hungary, years

Class	1	2	3	4	5	6	7
	<10%	<20%	<30%	<50%	<100%	<200%	>200%
1	188.7	124.8	254	390	725	2618.8	8168.8
2	1300	45.5	178.8	348	672	2551	8107
3	1620	320	43.5	228	590	2508	8068
4	1795	495	174	40	440	2435	8000
5	1920	622	302	130	58.8	2118	7710
6	2077.6	778	457.8	289	157	200	5780
7	2195	895	576	409	276	124	500

Table A.5.4.2.18. Mean first passage time matrix for Russia, years

Class	1	2	3	4	5	6	7
	<10%	<20%	<30%	<50%	<100%	<200%	>200%
1	20	160	318	497	1060	2900	4477
2	95	47	270	460	1020	2850	4420
3	144	60	125	370	936	2780	4350
4	200	120	130	140	739	2580	4160
5	290	210	227	1690	230	2030	3646
6	469	380	400	346	190	357	2110
7	617	530	550	498	340	247	150

Table A.5.4.2.19. Initial and ergodic distributions for Polish cities

	1	2	3	4	5	6	7
	<10%	<20%	<30%	<50%	<100%	<200%	>200%
Initial distribution	0.137	0.216	0.14	0.157	0.174	0.085	0.087
Ergodic distribution	0.017	0.038	0.02	0.029	0.1	0.156	0.64

Table A.5.4.20. Initial and ergodic distributions for Belarusian cities

	1	2	3	4	5	6	7
	<10%	<20%	<30%	<50%	<100%	<200%	>200%
Initial distribution	0.199	0.186	0.198	0.183	0.091	0.05	0.092
Ergodic distribution	0.56	0.12	0.16	0.11	0.03	0.008	0.015

Table A.5.4.21. Initial and ergodic distributions for Hungarian cities

	1	2	3	4	5	6	7
	<10%	<20%	<30%	<50%	<100%	<200%	>200%
Initial distribution	0.049	0.122	0.139	0.237	0.255	0.126	0.073
Ergodic distribution	0.053	0.22	0.23	0.25	0.17	0.05	0.02

Table A.5.4.22. Initial and ergodic distributions for Russian cities

	1	2	3	4	5	6	7
	<10%	<20%	<30%	<50%	<100%	<200%	>200%
Initial distribution	0.156	0.209	0.134	0.166	0.152	0.081	0.103
Ergodic distribution	0.497	0.212	0.08	0.07	0.043	0.028	0.067

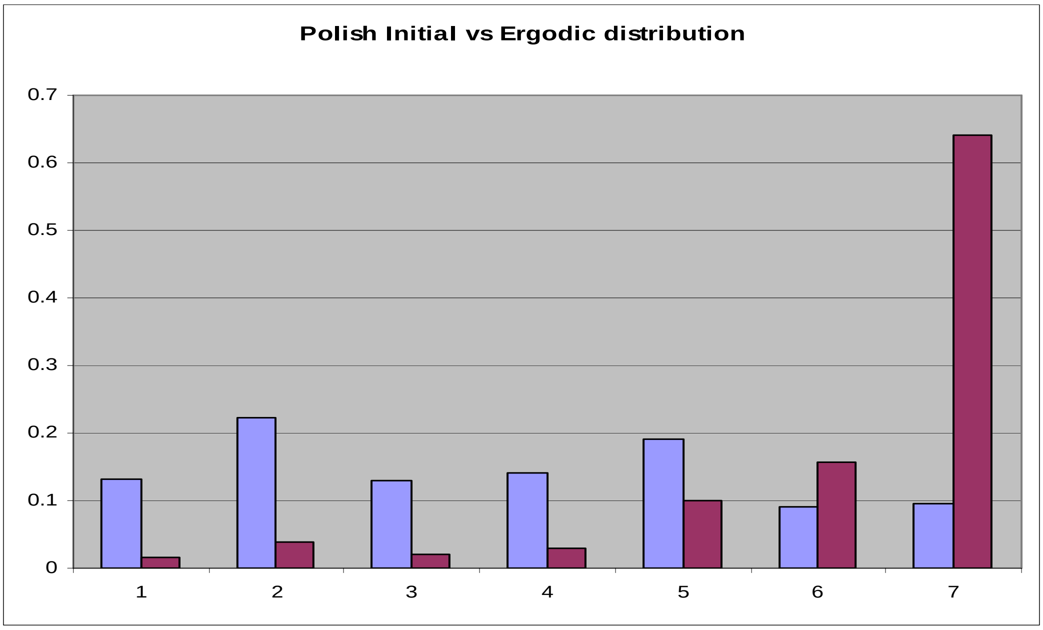
Table A.5.4.23 Initial vs ergodic distribution 1900—2001: Spain

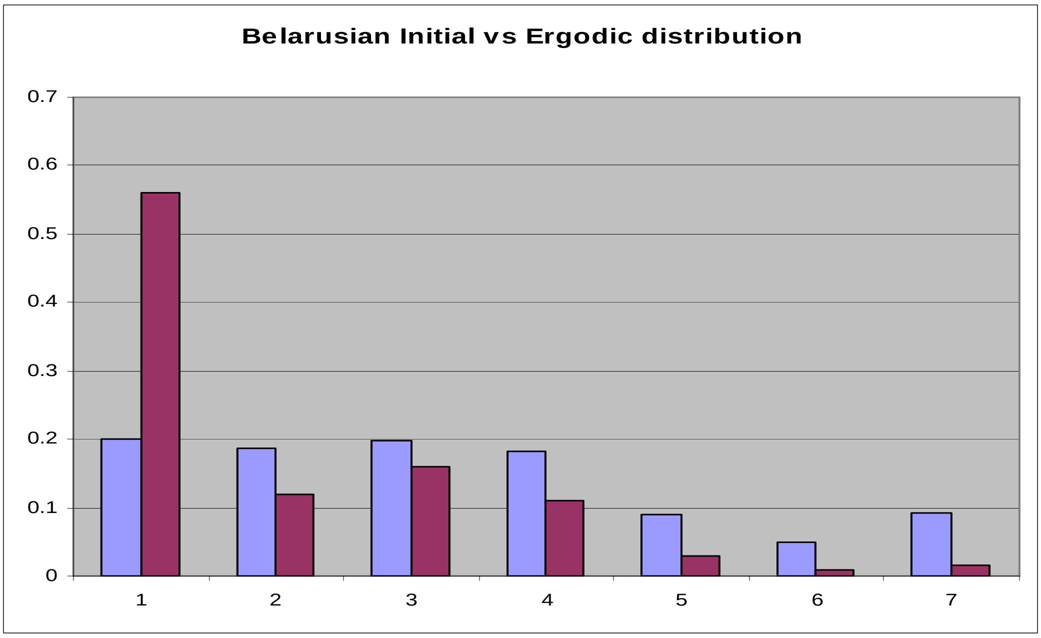
	1	2	3	4	5	6
	<20%	<50%	<80%	<135%	<185%	>185%
Initial distribution	0.356	0.243	0.143	0.118	0.044	0.098
Ergodic distribution	0.254	0.355	0.181	0.098	0.035	0.078

Table A.5.4.24. The values of kurtosis across countries

	Poland	Belarus	Hungary	Russia
Initial distr.	-0.40628	-1.98351	-0.98227	-0.00516
Ergodic distr.	5.84045	5.03177	-2.41139	4.18436
Difference	4.99726	6.80950	-1.64034	5.75212

Figure A.5.4.1 Initial vs ergodic distributions (Blue – Initial, Red – Ergodic).





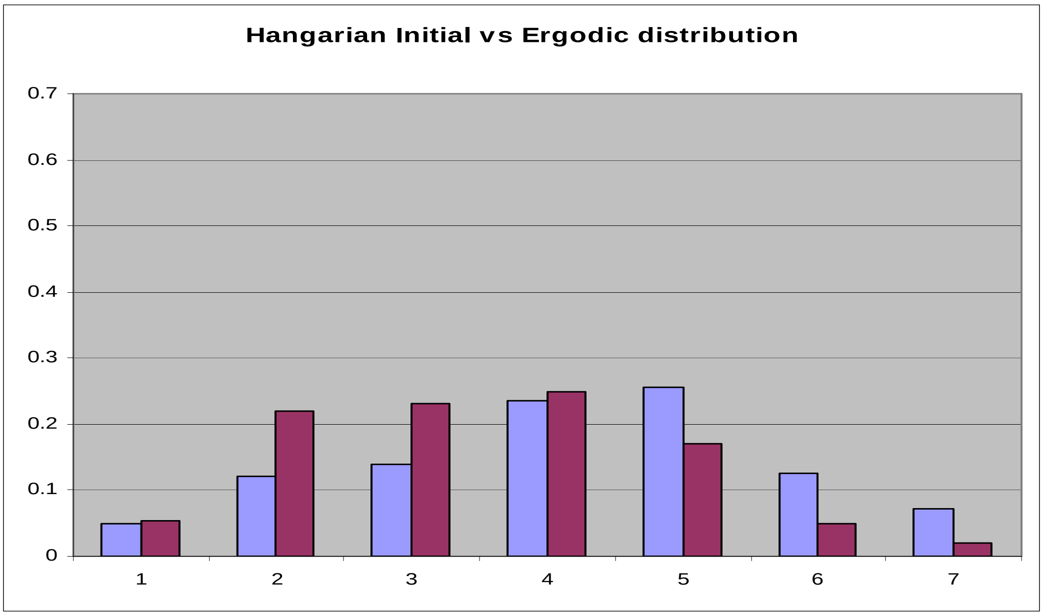


Figure A.5.4.2. Map of Belarus, by red is depicted growing cities, by blue is depicted vanishing cities

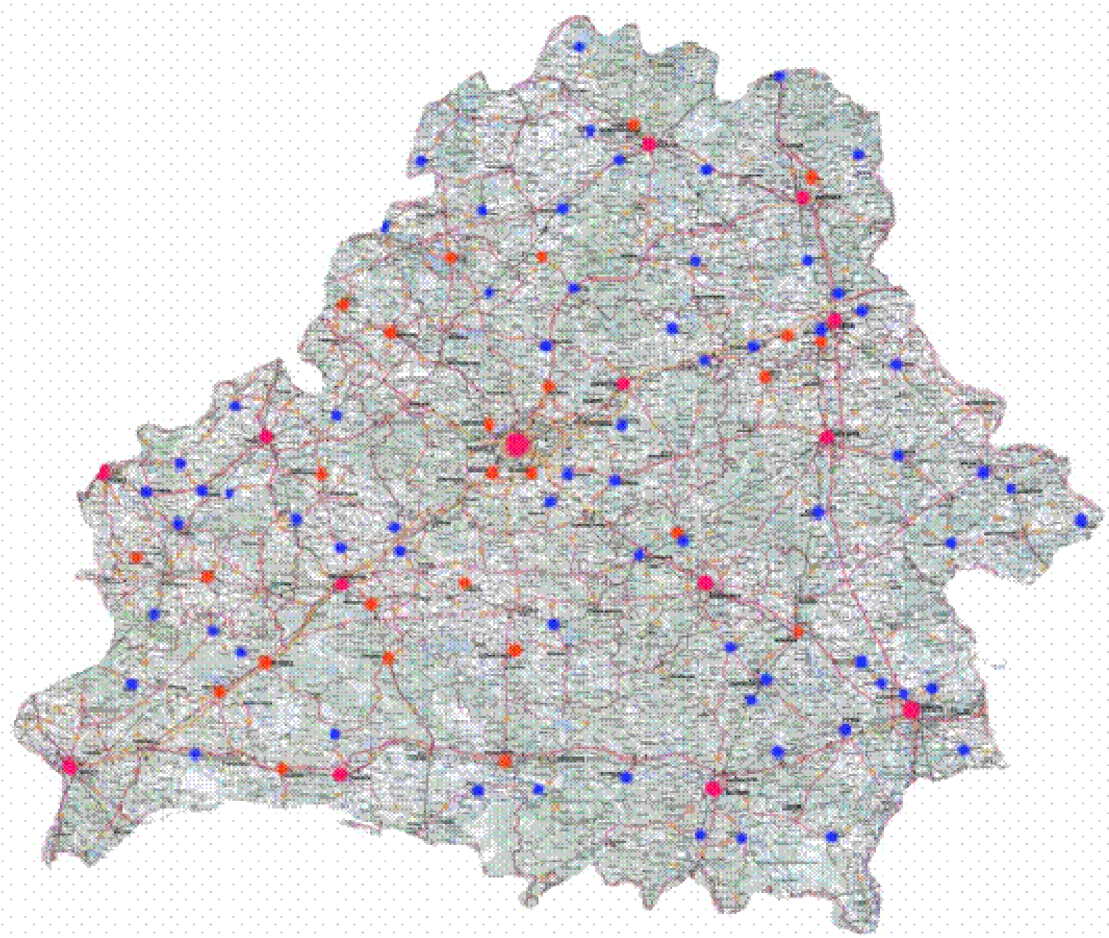


Table A5.5.1. *Model 1 results*

pareto_cons	Coef	Std. Err.	t	P>|t|	[95% Conf. Interval]	
gdpa	0,000366	7,05E-05	5,19	0	0,000227	0,000506
raila	0,065931	0,015812	4,17	0	0,034668	0,097195
telpc	0,001087	0,001053	1,03	0,304	-0,001	0,003169
mobpc	-0,0008	0,000224	-3,56	0,001	-0,00124	-0,00036
fri	-0,0059	0,00548	-1,08	0,283	-0,01674	0,004934
prim1	0,860976	1,907311	0,45	0,652	-2,91012	4,632068
prim5	-3,01251	1,156043	-2,61	0,01	-5,29821	-0,7268
ab_ratio	-4,3E-05	1,87E-05	-2,3	0,023	-8E-05	-6.04e-06
year	0,000413	0,001561	0,26	0,792	-0,00267	0,0035
_cons	0,51106	3,058127	0,17	0,868	-5,5354	6,55752
R-sq:	within	0.7406		sigma_u	0,423641	
	between	0.2170		sigma_e	0,042469	
	overall	0.1920		rho	0,99005	
	F(9,139)	44.09				
	corr(u_i, Xb)	-0.9630				

Table A5.5.2. *Model 2 results*

pareto_cons	Coef	Std. Err.	t	P>|t|	[95% Conf. Interval]	
gdpa	0,0001147	0,0000775	1,48	0,141	-0,0000386	0,000268
raila	0,0089764	0,0147515	0,61	0,544	-0,0201936	0,0381464
telpc	-0,004689	0,0011027	-4,25	0	-0,0068695	-0,0025086
mobpc	0,0021019	0,0046139	0,46	0,649	-0,0070217	0,0112255
fri	1,357783	1,570498	0,86	0,389	-1,74778	4,463334
prim1	1,357783	1,570498	0,86	0,389	-1,747767	4,463334
prim5	-3,782911	0,9720792	-3,89	0	-5,70513	-1,860691
ab_ratio	0,1360431	0,034285	3,97	0	0,0682469	0,2038392
year	0,0100561	0,001723	5,84	0	0,0066489	0,0134633
_cons	0,8426203	2,627306	0,32	0,749	-4,352696	6,037937
R-sq:	within	0.8289		sigma_e	0,0347403	
	between	0.1176		rho	0,9992547	
	overall	0.0859				
	F(9,139)	60.34				
	corr(u_i, Xb)	-0.9951				